BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 28, Number 2, April 1993

0273-0979/93 $1.00 + $.25 per page

Automorphisms and derivations of associative rings, by V. K. Kharchenko. Kluwer Academic Publishers, Dordrecht, 1991, xiv + 385 pp., $144.00. ISBN 0-7923-1382-8

It is ironic justice that Evariste Galois, whose brief and tormented life lasted barely twenty years, should have left us one of the most beautiful creations of modern algebra. Galois theory establishes a firm relation between field extensions and their automorphism groups, which not only elucidates the structure of fields and marks an important advance in the theory of equations, but has also contributed to a better understanding of groups; thus, it has helped to clarify the notion of normal subgroup. It is not surprising that many attempts at generalization have been made; the Galois theory of skew fields, mainly the work of H. Cartan and N. Jacobson, presents some analogy to the commutative case but does not go as far. In particular, it sheds almost no light on equations over skew fields, and the equality of the group order and the degree of the extension has to be modified by using the "reduced" group order, in which the number of inner automorphisms is replaced by the dimension of the space spanned. Now it is a small step to pass to simple Artinian rings by forming matrix rings.

On the other hand, from a prime ring satisfying suitable finiteness conditions (the Goldie conditions), one can obtain simple Artinian rings as quotients, and this suggests further extensions. There has been much work on automorphisms and derivations in various classes of rings in the last few decades, and the book under review is the first connected account of the subject (although a useful set of lecture notes by S. Montgomery appeared a dozen years ago). An initial chapter lays the groundwork on automorphisms and derivations. Almost the starting point is the Bergman-Isaacs theorem, which the author states and proves for semiprime rings: If a finite group G acts by automorphisms on a semiprime ring R with no additive $|G|$-torsion, then the fixed ring R^G is also semiprime. He goes on to describe the Martindale quotient ring construction; this is of particular interest for semiprime rings, when there is a natural sheaf structure over the space of idempotents of the centroid. The structure of primitive rings with nonzero socle is described in more detail, using a natural topology. These rings occur, for example, in Martindale's theorem on rings with a generalized polynomial identity (GPI): If R is a prime GPI-ring, then its central closure is primitive with nonzero socle, such that the endomorphism ring of a minimal right ideal is finite dimensional over its centroid.

Another key notion is the set of automorphisms of the ring R which are induced by inner automorphisms of the quotient ring. Its members are usually called X-*inner* automorphisms, after the author (Харченко), though nameless here. They allow a useful extension of many results proved for inner automorphisms in the first place. Both for automorphisms and for derivations, the question of linear or algebraic dependence turns out to be crucial. By using pth powers of derivations in characteristic p, the author ensures that the linearization process is reversible. It now becomes natural to consider identities involving automorphisms and derivations, briefly DA-*identities*. A system $\{f_\lambda\}$

of identities of a ring R is called *essential* if the ideal generated by the values of all the f_λ is the whole ring. Now the author is able to show that a ring with 1 and no additive torsion, which has an essential system of DA-identities, is a PI-ring.

The core of the book, Chapter 3, describes the Galois theory of prime rings. The main object of study is a prime ring with a reduced-finite group of automorphisms (that is, the reduced order is finite). For best results, the algebra of the group is assumed to be quasi-Frobenius. Moreover, the group is assumed to contain all inner automorphisms induced by invertible elements in its algebra, that is, it is an N-group (for E. Noether). Now the main result, for a prime ring with an N-group of automorphisms, provides a bijection between N-subgroups and intermediate subrings satisfying certain simple conditions. Similar results are established for prime rings of prime characteristic p and restricted Lie algebras of outer derivations. By means of a metatheorem on transferring Horn sentences from the stalks of a sheaf to its global sections, the author is able to extend these theorems to semiprime rings. The case of characteristic zero is of less interest because here there is only a weak relation between the constants of a finite-dimensional Lie algebra of derivations and the basic ring, but once restrictions are imposed, such as the finite dimensionality of the ring, the derivations turn out to be inner on the quotient ring and the whole problem is reduced to one of centralizers of finite-dimensional algebras, which can be treated by classical methods.

In the final chapter, on applications, the author attacks a number of disparate problems by the methods developed earlier in the book. It is more advanced in that more specialized results are assumed here without proof, but it is also the most interesting. Here we can find the author's theorem that the fixed elements of a free algebra under the action of a group of homogeneous automorphisms again form a free algebra, with a similar result for the constants of a Lie algebra of homogeneous derivations. The question of finite generation of the algebra of invariants is investigated, and Koryukin's recent work is presented. Now we do not generally have finite generation, in contrast to the commutative case (Hilbert-Nagata), but an analogous result is obtained once S-subalgebras are used, that is, homogeneous subalgebras admitting the symmetric group on their homogeneous components. This leads naturally on to a study of the relations of a ring with its fixed ring under a group of automorphisms, arising from the Bergman-Isaacs theorem. The effect of various conditions on the ring (primitivity, Goldie conditions, etc.) is studied, but no surprises emerge. A similar study is made for derivations and their rings of constants, and it is shown how these two cases, of automorphisms and derivations, can be combined by using the notion of a Hopf algebra.

It is clear that this book, by a member of the well-known Novosibirsk School of Ring Theory, who himself has made substantial contributions to the subject, will be welcomed by all specialists. The text is written with specialists in mind, for such notions as free product, weak algorithm, etc., are introduced without explanation, and trivial cases in proofs are often skipped. None of this, however, will present a serious obstacle to a patient (and persistent) reader; even the index, which is quite inadequate, and list of notation (absent) will cause only minor frustration. The real cause of complaint will be the wholly unsatisfactory translation, at best irritating but at worst seriously misleading. Much of it does not read like English at all, and many words are mistranslated, for example, simple for prime, conjunction for conjugation, boundary for bound, expanded for decomposed, directed for inductive, stempel for imprint, and many others. Definite and indefinite articles are constantly mixed up, and to add to all this there are numerous errors that careful proofreading should have caught; thus, the Queen's theorem turns out five pages later to be Quinn's theorem. The layout and type (word processor) also falls well short of a satisfactory standard. If properly typeset (or even done on a decent word processor), the book could have had a pleasing typeface and significantly less pages.

P. M. COHN
UNIVERSITY COLLEGE LONDON

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 28, Number 2, April 1993

0273-0979/93 $1.00 + $.25 per page

An introduction to wavelets, by Charles K. Chui. Academic Press, New York 1992, x + 264 pp., $49.95. ISBN 0-12-174584-8

Ten lectures on wavelets, by Ingrid Daubechies. CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, 1992, 357 pp., ISBN 0-89871-274-2

It would have been difficult to review the books written by Ch. Chui and L. Daubechies without describing the goals of the "wavelet venture". Wavelets were discovered in a signal processing context much before mathematicians became aware of their existence, and for this reason the first four sections of this review will be mostly science oriented. But an amazing discovery of the "wavelet revolution" has been the identification of the algorithms proposed by the signal processing community with some tools developed by the famous Calderón-Zygmund school. Sections 5–7 will be devoted to these relations. Chui's book will be analyzed in §8, Daubechies's in §9, and in §10 we give our personal feelings about the relevance of wavelets in science and technology.

1. WAVELET ANALYSIS VERSUS WINDOWED FOURIER ANALYSIS

Wavelet analysis can be defined as an alternative to the classical *windowed Fourier analysis.* In the latter case the goal is to measure the local *frequency* content of a signal, while in the wavelet case one is comparing several *magnifications* of this signal, with distinct *resolutions.* Wavelet analysis is related to *multiresolution analysis* in §6. The building blocks of a windowed Fourier analysis are sines and cosines (waves) multiplied by a *sliding window.* In a wavelet analysis the window is already oscillating and is called a *mother wavelet.* This mother wavelet $\psi(x)$ has a compact support (or a rapid decay at infinity), is smooth, and satisfies the fundamental condition $\int_{-\infty}^{\infty} \psi(x)\,dx = 0$, which means that in some weak sense $\psi(x)$ is oscillating. Moreover, the mother wavelet should also satisfy a technical but crucial condition, which will be revealed in §5. The mother wavelet is no longer multiplied by sines