

BOOK REVIEW

Scientific computing and differential equations—An introduction to numerical methods, by Gene H. Golub and James M. Ortega. Cambridge Univ. Press, Cambridge and New York, 1992 and 1981, xi+337 pp., \$49.95. ISBN 0-12-289255-0

This book is a textbook for a first course in numerical analysis that focuses on ordinary and partial differential equations. Many other topics that usually are included in a first course in numerical analysis, such as the solution of systems of linear and nonlinear algebraic equations, interpolation and approximation, numerical integration, etc., are treated as they arise in the development of methods for the solution of differential equations. This unusual organization of the material is intended to provide greater motivation for the students than might be the case in a more conventional numerical analysis course.

The reader is assumed to have had calculus, first courses in computer programming and differential equations, and at least some linear algebra. It is expected that the book can be covered in two semesters by students with a minimum background and no previous numerical analysis courses; however, at the University of Virginia, the material is regularly covered in one semester in classes where the students have stronger backgrounds.

The book starts with an interesting discussion of the subject of scientific computing and its relation to mathematics, mathematical modeling of problems in science and engineering, numerical analysis, and computer science. The process of obtaining numerical solutions is described with emphasis on rounding errors, discretization error, computational efficiency, and computational environments.

The authors then take up one-step and linear multistep methods for solving initial-value problems involving ordinary differential equations. The treatment includes a discussion of stability and instability as well as stiff equations. Then they discuss finite difference methods for solving boundary value problems for ordinary differential equations. The use of Gaussian elimination for solving tridiagonal systems is described for the solution of the discretized problem.

As an alternative to finite difference methods, projection methods, such as collocation, Galerkin's method, and methods based on the use of splines, are considered. This motivates a discussion of numerical integration methods including Newton-Cotes as well as Gaussian integration and Romberg integration.

A more general treatment of the problem of solving linear systems is given next. The approximation of functions of one variable by least squares is considered and solutions based on the use of the normal equations and on the use of orthogonal polynomials are described. The solution of general systems of linear equations by

Gaussian elimination and by LU factorization is then described. Other factorization methods such as QR factorization, based on Householder and Givens transformations and on Cholesky factorization, are also treated.

Some discussion is given to the solution of a single nonlinear equation and systems of nonlinear equations. For systems of equations Picard iterations, Newton's method, and continuation methods are considered.

Matrix eigenvalue problems are discussed next. The methods considered include the QR method, Householder's method, and Givens's method, as well as iterative methods such as the power method, the inverse power method, and Lanczos's method.

Partial differential equations take up the remainder of the book. Time-dependent equations, such as the heat equation $u_t = u_{xx}$ and the wave equation $u_{tt} = u_{xx}$, are considered. In addition to Fourier methods, explicit and implicit finite difference methods are discussed and analyzed in terms of accuracy and stability. The method of lines is also treated.

The final topic, which is entitled "The Curse of Dimensionality", is devoted to methods for solving steady-state and time-dependent problems in two and three space dimensions. The Peaceman-Rachford alternating direction implicit method is among the methods considered for solving time-dependent problems, where implicit methods are used, and also for solving steady-state problems. Other iterative methods considered include the Jacobi, Gauss-Seidel, SOR, and conjugate gradient methods.

The emphasis of the book is on mathematical methods rather than numerical algorithms and software; however, exercises, which require algorithms and computer programs, are given. One advantage of this treatment is that the book is not diluted with a mass of detail. The discussion is very clear, and the notation and format of the book are very attractive.

This book should be very valuable not only as a textbook for the beginning student of numerical analysis but also for a more advanced student in that it provides a new perspective and better understanding of the entire subject.

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