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Finite soluble groups, by Klaus Doerk and Trevor Hawkes. De Gruyter Expositions in Mathematics, vol. 4, de Gruyter, Berlin and New York, 1992, x + 891 pp., $148.00. ISBN 3-11-012892-6

This is an account of the development of the theory of finite soluble groups over the last thirty years, concentrating on "those parts of the subject where a coherent and unified body of knowledge has emerged": the theory of Schunck classes, formations, and Fitting classes of soluble groups with their associated subgroups. My first impression of the book was of its size. It contains almost 900 pages, including indices and appendices. It is written carefully and clearly and will become, as the authors hope, "a basic reference [for soluble groups], a text for postgraduate teaching, and ... a source of research ideas and techniques". The book is about *finite* groups, but of course infinite groups turn up at times, for example, when working with groups associated with infinite fields or with Fitting classes. Again, the book is about *soluble* groups, but the authors often explore "insoluble territory to see where the soluble theory leads".

A group G is said to be *soluble* if it has a series of subgroups $G = G_0 > G_1 > \cdots > G_n = \{1\}$ such that G_i is a normal subgroup of G and G_{i-1}/G_i is abelian for all i.

The two fundamental results on which the theory of finite soluble groups has been built are the theorems of Sylow and Hall. Sylow's theorems state that, for a prime p and a finite group G of order $p^a m$ with p not dividing m, G has a unique conjugacy class of subgroups of order p^a, now called *Sylow p-subgroups* of G, and any p-subgroup of G is contained in a Sylow p-subgroup. In 1928 Hall [6] published a far-reaching generalisation of Sylow's theorems for soluble groups. Let π be a set of prime numbers, and let G be a finite group. The order $|G|$ can be factorised as nm where n is a π-number (that is, all its prime factors lie in π) and m is a π'-number (none of its prime factors lie in π). A subgroup whose order divides n is known as a π-subgroup, and a subgroup of order n is now known as a *Hall π-subgroup*. What Hall proved was that if G is soluble, then G has Hall π-subgroups, they form a single conjugacy class, and any π-subgroup is contained in a Hall π-subgroup. In 1937, nine years later, Hall went on to show that this fact actually characterises finite soluble groups in the sense that if a finite group G has Hall π-subgroups for every set of primes (of course, only subsets of the primes dividing $|G|$ are relevant), then G is soluble.

To make the book accessible to students and to colleagues who are not specialists in the area of finite soluble groups, the authors begin with two large introductory chapters containing an extensive collection of basic results about finite groups and their matrix representations. These chapters give an excellent summary of the definitions and fundamental theory used in the main chapters. If the proof of a result is available in [8] or [9], then the proof is omitted and a reference to [8, 9] is given; if not, the result is proved. Thus, although the reader needs a basic knowledge of the theory of finite groups and their representations, these two introductory chapters provide the necessary prompts to the memory.

Chapter I, that is, the first chapter after the introductory chapters, begins with a careful elementary proof of the theorem of Burnside that a finite group of order $p^a q^b$, with p and q prime, is soluble. Burnside's theorem is then used in the proof of the theorems of Hall. At this point the authors discuss other generalisations of Burnside's theorem in the context of finite soluble groups and in the wider context of general finite group theory. They mention quite recent results, some using the finite simple group classification, give several references to the research literature, present and discuss a related open problem, and end the section with a set of exercises. Such discussions and exercise sets are a feature of the book. The rest of the chapter explores Hall systems and their normalisers and pronormal subgroups of groups.

The next chapter gives a general discussion of closure operations on classes of groups and introduces Schunck classes, formations, Fitting classes, and varieties of groups. Chapter III develops the theory of Schunck classes, introducing projectors and covering subgroups. It begins with a historical introduction in which significant discoveries in the area are presented as a development of a generalised Sylow theory. It traces the 1961 discovery of Carter subgroups (self-normalising nilpotent subgroups of soluble groups, in [1]) and Gaschütz's extension of them in [5] to covering subgroups of saturated formations, through to recent work of Förster [2–4], which encompasses insoluble and soluble groups. The discussion at this point is important for giving the nonexpert reader a good perspective of the role played by these classes of groups in the development of the theory of finite soluble groups. The remaining chapters deal with further theory of Schunck classes and formations and with the theory of Fitting classes.

Another good feature of the book is the collection of well-chosen examples presented informally and in detail. These appear at crucial stages to illustrate important aspects of the theory.

Overall I find the book to be well written and readable. Because of its size and the problems this might cause in finding particular topics or definitions or explanations of unusual notation, it is particularly important that there should be a good index and list of contents. I was pleased to find these features, in addition to a list of symbols, an index of names, and an extensive bibliography. Klaus Doerk and Trevor Hawkes are to be congratulated on their achievement.

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Topics in non-commutative geometry, by Yuri I. Manin. M. B. Porter Lecture Series, Princeton University Press, Princeton, New Jersey, 1991, 164 pp. $35.00. ISBN 0-691-08588-9

Noncommutative geometry emerged as a branch of mathematics at the end of the Grothendieck era. Originally its goal had been to geometrize arbitrary noncommutative rings, i.e., first to associate to a noncommutative ring a "noncommutative spectrum" by extending the construction of the prime spectrum of a commutative ring and then to "glue" these spectra into a "noncommutative scheme". The glueing problem turned out to be very difficult, and it does not seem to have been solved in a way meeting initial expectations. A reason is that general noncommutative spectra are not sufficiently functorial in order to be localized (or glued together) in the usual fashion. However, this is an active field; for an essential recent development see [1]. It is remarkable that, nevertheless, noncommutative geometry has made outstanding progress and has in the last decade been constantly among the "hottest" subjects in pure mathematics as well as in mathematical physics. This is because since the 1970s the very idea of noncommutative geometry has become much more complex. Today this is one of those fascinating subjects which ignore customary interdisciplinary boundaries and where, for instance, algebraic geometry in characteristic p and Feynman integrals live together.