also quite good. What many people will find difficult to swallow, however, is the price of the book—$181 list, $109 for AMS members.

ACKNOWLEDGMENTS

The reviewer thanks the author for explaining the history of the publication of this book and G. Kamvosoulis and K. Mederer for helpful remarks.

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GERALD W. SCHWARZ
BRANDEIS UNIVERSITY
E-mail address: schwarz@math.brandeis.edu

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 29, Number 2, October 1993

0273-0979/93 $1.00 + $.25 per page

Kolmogorov spectra of turbulence I: *Wave turbulence*, by V. E. Zakharov, V. S. Lvov, and G. Falkovich. Springer Verlag Series in Nonlinear Dynamics, Springer-Verlag, New York, 1992, xi + 264 pp., $99.00. ISBN 3-540-545336

The theory of fully developed turbulence in fluids and other continua is of enormous practical importance and great theoretical interest. It has seen much progress in recent years: Dynamical system theory has provided credible models of the inception of turbulence and has justified the use of probabilistic methods; the fractal character of turbulence has been clearly recognized; important progress has been made in the analysis of the underlying equations. Most importantly, developments in numerical methods and increases in the power of computers have made possible some beautiful and illuminating calculations and have elicited novel modelling techniques. Despite this progress, the problem of making reliable predictions remains mostly open, and the available theory is fragmentary; the extent of our ignorance is particularly baffling in a problem that belongs to classical mechanics and that can be described by simple-looking differential equations known for more than a century.

Nothing illustrates better the way in which turbulence is suspended between ignorance and light than the Kolmogorov theory of turbulence, which is both the cornerstone of what we know and a mystery that has not been fathomed. In the late thirties the great mathematician Kolmogorov, in collaboration with a brilliant group of hydrodynamicists and meteorologists, recognized the "universality" of the small scales of turbulence, i.e., their independence from the contingencies of whatever stirred the fluid, and drew important conclusions about those "universal" scales. The best known is probably the Kolmogorov "5/3 law", which asserts that, on scales much smaller than the stirring scales yet not so small that dissipation is important, the energy spectrum, a measure of the energy distribution between wave numbers k, is proportional to $k^{-5/3}$. The same spectrum thus appears in the sun, in the oceans, and in man-made machinery. The 5/3 law is well verified experimentally and, by suggesting that not all scales must be computed anew in each problem, opens the door to practical modelling. It is the starting point of most analysis [2].

Kolmogorov's theory is not a collection of theorems; it is a brilliant piece of dimensional analysis guided by physical intuition. Its major premise is that energy cascades, as in a waterfall, from large to small scales, creating the spectrum along the way. Kolmogorov's reasoning was assailed almost as soon as it appeared; in a famous footnote to his textbook of fluid dynamics Landau pointed out that Kolmogorov's analysis did not square with "intermittency", i.e., with the spatial spottiness or unevenness of turbulence. As a result, various corrections to Kolmogorov's theory have been proposed, including a misguided effort by Kolmogorov himself and a variety of analyses based on fractal characterizations of Kolmogorov's range. None of the corrections to Kolmogorov's result has withstood careful experiment and calculation, but the corresponding analyses have certainly cast a shadow on its derivation.

Even more fundamental questions about the derivation have been raised. Kolmogorov's spectrum often appears in problems where his assumptions clearly fail. The idea of an irreversible cascade that creates the spectrum has been undermined; there is overwhelming evidence that energy goes up and down the ladder of scales in nearly equal measures [3]. Organized structures spanning many scales appear and affect the flow, like boulders in a waterfall, in a way that Kolmogorov's argument does not allow for. The 5/3 law can now be derived in many ways, often under assumptions that are antithetical to Kolmogorov's. Turbulence theory finds itself in the odd situation of having to build on its main result while still struggling to understand it.

It has been argued that one could fruitfully study Kolmogorov's argument in the simpler setting of weak (or "wave") turbulence, i.e., in systems that allow waves to interact but are not too nonlinear. It may be that organized structures play no role under these circumstances; interactions between waves are not without similarity to interactions between particles and can be described by "kinetic" equations; it may be possible to relate the Kolmogorov spectrum to specific properties of these kinetic equations, if the uniqueness of the kinetic representations and of their solutions can be displayed. The systems in which weak turbulence occurs have an intrinsic interest, and Zakharov's work in particular has original and unexpected aspects; a clear exposition of what is known in this area would be very welcome.

The book at hand aims to provide such an exposition. Its table of contents is an accurate list of what should be told. However, the book itself is opaque. The English requires extensive editing; the notation is often obscure, at least for people not educated in the former Soviet Union; the explanations are sketchy; and even accounts of elementary results can be incomprehensible (e.g., the introductory account of Hamiltonian systems). A Hamiltonian formalism is used in the analysis of Kolmogorov's argument, in which dissipation is very important; the paradox is not acknowledged, let alone resolved. The difference between the exponent in Kolmogorov's law obtained here and the one obtained in a similar way by Kraichnan [1] is not explained; Kraichnan found that to recover Kolmogorov's result he needed a Lagrangian correction of the type relegated here to another volume. In addition, the book is very heavily biased toward work in the former Soviet Union. Of 163 references, only 35 are to work done elsewhere, and even these references are often to papers of historical interest—some from the nineteenth century—or to elementary textbooks. As a result, important insights available in the Western literature are glaringly absent. The present book may be a useful guide to the Soviet literature for experts in this field, but it is hard to recommend to a broader audience.

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Alexandre J. Chorin
University of California at Berkeley
E-mail address: chorin@math.berkeley.edu

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 29, Number 2, October 1993

0273-0979/93 $1.00 + $.25 per page

Sphere packings, lattices and groups, second ed., by J. H. Conway and N. J. A. Sloane. Grundlehren der Mathematischen Wissenschaften, bd. 290, Springer-Verlag, New York and Berlin, 1993, xliii+679 pp., $69.00. ISBN 0-387-97912-3 and ISBN 3-540-97912-3

In 1611 Johann Kepler published an essay "On the Six-Cornered Snowflake"; see [10]. He explains how equal rhomboid dodecahedra will fit together to fill space. He goes on to explain how the loculi of pomegranates obtain their rhombohedral shapes, arguing that a fixed number of initially round loculi swell within the tough skin, first to form a densest possible arrangement of spheres with each sphere touching twelve others, and then swell further, pressing against each other to take the shape of rhomboid dodecahedra. The sphere packing that he claims to be densest is just the familiar arrangement used to pile cannon balls or oranges. A similar claim to have found the densest arrangement of spheres