(and which Marsden treats in detail) is that of the "falling cat". How is it that a cat, dropped upside down from an appropriate height, will twist so as to land on its feet? It turns out that this is a subtle problem with an elegant solution, and it is part of the more general problem of the dynamics of deformable bodies and feed-back control.

Marsden's book is not a textbook but a series of lectures on various aspects of symmetry in dynamics. There is enough background to make the book reasonably self-contained, and there is a very thorough bibliography and an indication of where to go to pursue any of the more specialized topics. The style is readable and stimulating.

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Tube domains and the Cauchy problems, by Simon Gindikin. Translations of Mathematical Monographs, vol. 111, American Mathematical Society, Providence, RI, v + 132 pp., \$78.00. ISBN 0-8218-4566-7

This book, which is part of the author's thesis, deals with research from the early sixties carried out by a circle of former students of Gelfand. There are two parts. The first one deals with a specific class of partial differential operators, the second with certain generalized gamma functions associated with homogeneous cones.

The classical classes of differential operators—elliptic, parabolic, and hyperbolic—are all of second order, and their importance stems from physics. The study of higher-order constant coefficient operators for their own sake was made possible by the theory of distributions. In the middle fifties, three classes of such operators P had been characterized by intrinsic properties as follows: elliptic (all solutions of $Pu = 0$ analytic), hypoelliptic (all such solutions infinitely differentiable), and hyperbolic (fundamental solution with support in a cone). In all cases there are corresponding properties of the characteristic polynomials.

The first part of Gindikin's monograph is a second generation effort in the same direction. The starting point is a separation of variables in time t and space $x \in R^n$ and the corresponding Cauchy problem. Let $D_t = \partial/i\partial t$ and $D_x = \partial/i\partial x$ be the imaginary gradients so that $P(\tau, \xi)$ is the characteristic polynomial of $P(D_t, D_x)$. The author considers a class of operators for which $P(\tau, \xi)$ does not vanish in some tubular region

$$T:\ \operatorname{Im}\tau < -\chi(\operatorname{Im}x) - \text{const}$$

where χ is a fixed, finite convex function. Operators in this class have inverses given by the Fourier-Laplace transform and operate on classes of functions whose size in the x-directions is controlled by the dual of χ. The class itself is invariant under complex translations. With the added condition that the highest power of τ appears with a constant, the class can be characterized by unique solvability of certain Cauchy problems. Extensions to variable coefficient equations are also given.

The second part of the book, which is only weakly connected with the first part, is perhaps more interesting. It rests on a theorem by E. B. Vinberg (Trans. Moscow Math. Soc. **12** (1965)) which gives the structure of open, proper homogeneous convex cones and is intimately connected with work by Piatetskii-Shapiro and others on the same problem for convex domains.

Vinberg's structure theorem says that the automorphism group G of a convex open cone C with a transitive automorphism group can be written as the product KT of a maximal compact subgroup K and a solvable linear group T which has only the identity in common with K. Moreover, all elements of T can be represented as generalized triangular matrices t of order l with positive diagonal elements. The other elements t_{ik} with $1 \le i \le k \le l$ belong to real linear spaces R_{ik} of dimension n_{ik} for which there are commutative bilinear multiplications $R_{ik} \times R_{kj} \to R_{ij}$ given by the group structure. The cone can then be represented by similarly constructed generalized symmetric matrices $x = tt^*$. Associated with the representation $x = tt^*$, there are l characters $\chi_i(x) = t_{ii}^2$ which are rational functions of x (quotients of determinants). When written in canonical form, the dual cone with elements $x^* = t^*t$ has characteristic dimensions $n_{ik}^* = n_{l+1-i,l+1-k}$.

There is a universal construction of homogeneous cones increasing the order l by 1. When $n_{ik}^* = n_{ik}$, the cone C is said to be symmetric. The existence of nonsymmetric cones was discovered in connection with the work mentioned above.

In the examples to follow, all cones are symmetric, and for the first two the formula $x = tt^*$ is classical: the cones of positive symmetric matrices, $(n_{ik} = 1)$, or positive hermitian matrices, $(n_{ik} = 2,\ i \neq k)$, and the Lorentz cone in $n+2$ dimensions in which case t has order 2 and t_{12} is a vector u with n components. The corresponding x has the components

$$x_{kk} = t_{kk}^2, \qquad x_{12} = (u, u).$$

In the last part of Gindikin's book the classical formula for $l = 1$,

$$\xi^{-\alpha} = \int_{x>0} e^{-x\xi} x^\alpha \, dx/x\Gamma(\alpha)$$

where $\xi > 0$, $\operatorname{Re}\alpha > 0$, is extended to homogeneous cones C. In this case the left side with $\xi \in C^*$ is a product

$$\prod_1^l \chi_i(\xi)^{\alpha^*}, \qquad \alpha_i^* = \alpha_{l+1-1}.$$

On the right the product $x\xi$ is a duality between C^* and C, and the rest of the integrand has the form

$$F_C(\alpha, x) = x^\alpha \mu(dx)/\Gamma_C(\alpha)$$

where $\mu(dx)$ is a certain invariant measure and the denominator is a product of gamma functions $\Gamma(\alpha_i - m_i/2)$ where m_i is the sum of the n_{ji} for $j > i$.

As when $l = 1$, the formula extends to complex matrices ξ with real parts in C, the matrices $i\xi$ forming a generalized Siegel upper halfspace. The distribution $F_C(\alpha, x)$ is an entire function of α which is the identity when $\alpha = 0$. When $\alpha_i = m_i/2$ for just one i, the support of $F_C(\alpha, x)$ is contained in the boundary of C; and in some cases the support is even smaller. If, for instance, C is the cone of hermitian matrices $x \geq 0$, the support of $V_C(\alpha, x)$ is the set of matrices of rank i when $\alpha_j = i$ for all $j \geq i$.

When $l = 1$, differentiation lowers the parameter α by one. In the general case, when such operators exist, they have fundamental solutions with support in C. When C is the cone of symmetric $l \times l$ hermitian matrices $x \geq 0$ and all $\alpha_i = \alpha$ are equal, the operator $\det(\partial/\partial x_{ik})$ has this property so that $F_C(1, x)$ is a fundamental solution with support in the set of nonnegative matrices of rank ≤ 1. This example, known before, may have motivated the last part of the book.

As Gindikin himself says, his book reflects interests of the early sixties. There is a considerable overlap between the second part and earlier translations of Russian Mathematical Surveys, no. 19 (1964) by Gindikin, no. 12 (1965) by Vinberg, and no. 16 (1968) by Gindikin and B. R. Vainberg.

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Multidimensional integral transformation, by Ya. A. Brychkov, H. J. Glaeske, A. P. Prudnikov, and Vu Kim Tuan. Gordon & Breach Science Publishers, Philadelphia, PA, 1992, xiii + 386 pp., \$76.00. ISBN 2-88124-839-X

The concept of an integral transformation is familiar. If a function f is defined on a space Ω and the function $\tilde{f}$ is defined by an equation of the form

$$\tilde{f}(y) = \mathscr{T}[f(x); x \to y] = \int_\Omega f(x)K(x, y)\,dx \tag{1}$$

where the function K is defined on $\Omega \times \Omega^*$, the usual practice is to call the operator $\mathscr{T}: \Omega \to \Omega^*$ an *integral transformation* and the function $\tilde{f}$ an *integral transform*, though in this volume there is a certain confusion and the terms are regarded as interchangeable.

In Chapter IX of his treatise [1], in discussing the diffusion of heat in an infinite solid, Fourier introduced—but hardly could be said to have proved!—theorems concerning double integrals which were later interpreted as theorems involving *Fourier transforms.* Similarly what we now call *Laplace transforms*, which in the notation of (1) have Ω equal to the positive real line and $K(x, y) = e^{-xy}$, may be seen to have origins in Chapter 1 of Part 2, Volume 1 of Laplace's treatise on probability [2].