Chapter III of this book is devoted to a study of *residue currents* associated to complete intersections of positive dimension. The theory here, due to Coleff, Herria, and others, has numerous applications to local analytic geometry generalizing many of the properties of zero-dimensional complete intersections. The analysis is naturally somewhat more involved and requires the careful definition and use of principal values. In the case of codimension-one hypersurfaces contact is made with the "tube-over-cycle" residue map used by J. Leray in his study of fundamental solutions of hyperbolic partial differentiality equations.

In conclusion, what may we conclude about residues in several complex variables? Although the subject did not develop in this way, the monograph provides a convincing demonstration that residues may be used in an effective manner to develop much of local analytic geometry. Although they are less successful in dealing with ideals that are not local complete intersections, this is more than offset by the efficiency and eloquence with which residues apply to the wide variety of topics included in this book. One could well imagine an appealing introductory textbook in several complex variables that takes the residue (1) as its point of departure and for which the curent book would provide a valuable source of interesting applications.

A number of topics in global analytic and algebraic geometry are not covered in the monograph under review, so the reader may not get a sense of the extent to which residues may eventually have a significance in higher dimension comparable to their profound use in compact Riemann surface thoery. Among topics we would mention are the relationship between residues and Hodge theory and the various extensions of Abel's theorem. Regarding the latter, there is a recent paper by Henkin (G. M. Henkin, *The Abel-Radon Transform And Several Complex Variables*) in which residues are used via the "Abel transform" to give a far-reaching extension of the classical Abel's theorem, which in its original formulation was based on the elementary observation that $\mathrm{Res}_0\, \omega(\xi)$ is meromorphic in a parameter ξ if $\omega(\xi)$ itself is.

In summary, the present volume is useful and well presented, although a substantially expanded index would have been helpful; however, the reviewer would have preferred more emphasis on global results illustrating the priniciple that "$\sum \mathrm{Res} = 0$ is an analytic manifestation of compactness", a principle that has yet to be fully explored in applications.

PHILLIP A. GRIFFITHS
INSTITUTE FOR ADVANCED STUDY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 30, Number 2, April 1994

0273-0979/94 $1.00 + $.25 per page

Introduction to multidimensional integrable equations, by B. G. Konopelchenko. Plenum Publishing, New York, 1992, x + 292 pp, $85.00. ISBN 0-306-44220-5

The field of soliton theory and the related methods of solution of the underlying nonlinear evolution equations, referred to as the Inverse Scattering

Transform (IST), have been intensively studied for over twenty years. There are many nonlinear equations of physical significance which fit into the theory, the best-known examples being the Korteweg-de Vries (KdV), nonlinear Schrödinger (NLS), and sine-Gordon equations. These equations as well as many others which are $(1 + 1)$-dimensional (one space-one time) are special in that their exact solution, corresponding to initial data on the infinite line which decay sufficiently rapidly, reduces to a study of linear equations. The essential methods were developed in the 1970s. In the seventies it was known that certain $2 + 1$ equations also had properties which suggested that the IST techniques would be applicable. The prototypical equations are the Kadomtsev-Petviashvili (KP) and Davey-Stewartson (DS) equations, which arise in physics and are $2+1$ extensions of the KdV and NLS equations. Precisely how to solve these and related multidimensional systems is a central topic of this book.

In Chapter 1 a brief review of the IST method and related considerations is outlined. The solution process for $1 + 1$ problems is outlined, and indications of what will be covered in later chapters is given. For example, in pages 16–18 the solution of the KdV equation is described, and important results are summarized. Most likely readers will need to supplement this part of the text with any one of a number of books which describe the IST method for $1 + 1$ problems in considerable detail (this book has numerous references—over 700 of them—which will aid, though they might exhaust, the interested reader). The critical aspect of $1+1$ problems is that their solution is intimately connected to the study of local Riemann-Hilbert (RH) boundary value problems. In Chapter 2, probably the key chapter in the book, the methods applicable to $2+1$ problems are explained primarily in terms of the prototypes, KP and DS. The main new features of $2 + 1$ problems, corresponding to decaying data in the plane, are derived. Specifically, the reader finds out that, depending on the signs of the terms in KP or DS, analysis leads to the study of either nonlocal RH problems or DBAR problems. DBAR problems in complex analysis have been intensively studied by mathematicians. The application of these ideas in nonlinear wave problems is undoubtedly notable. The point of view here is one of explaining the methods and not going into the detailed functional analysis—often typical of applied mathematics. In recent years much of (but not all) the necessary functional analysis has been developed which has put the methods described in this book on a reasonably firm footing.

Chapters 3 and 4 contain a selection of techniques either to obtain special solutions, develop insight into the structure of the hierarchies of equations using algebraic analysis (sometimes referred to as the "tau function" approach), or understand the underlying symmetry structure of the equations. In Chapter 5 a discussion of the solution of higher-dimensional nonlinear equations is considered. The well-known prototype here is the four-dimensional self-dual Yang-Mills (SDYM) system, and methods to obtain solutions corresponding to specific gauge choice (SU(2)) are given. Actually recent work (not described in this book) has shown that the SDYM system, with the freedom of gauge heavily used, is actually a "master system" from which all of the well-known soliton equations can be obtained, by reduction, as special cases. On the other hand, the SDYM system seems to be in a very small class of integrable systems of high dimension (beyond the three-dimensional problems described earlier). The author explains in Chapter 5 why the scattering theory associated with multidimensional problems puts severe constraints on the evolution of data and why this is a serious obstacle to finding flows which will be compatible with linear systems for which the IST technique can be implemented.

The broad field of integrable systems is now enormous. No book could possibly cover all of the important aspects. This book carves out a piece of territory, multidimensional integrable equations and aspects of their solutions, especially those corresponding to decaying initial data, employing the IST method. The book can serve a reader well in many ways. The methods are explained, with lots of references given. Hence interested researchers can access the necessary external papers and books for any required material that is not contained in this book. The book would be a useful addition to a library's collection in a field of study which continues to expand rapidly.

MARK J. ABLOWITZ
UNIVERSITY OF COLORADO AT BOULDER
E-mail address: markjab@newton.colorado.edu

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 30, Number 2, April 1994

0273-0979/94 $1.00 + $.25 per page

The theory of semirings (with applications in mathematics and theoretical computer science), by Jonathan S. Golan. Pitman Monographs & Surveys in Pure and Appl. Math., vol. 54, Longman Scientific & Tech., Essex, 1992, 318 pp., $140.00. ISBN 0269-3666

There are many concepts of universal algebras generalizing that of an associative ring $(R, +, \cdot)$. Some of them—in particular, nearrings and several kinds of semirings—have proved very useful with respect to their various applications. Nearrings arise from rings by canceling either the axioms of left or that of right distributivity and allowing $(R, +)$ to be also a noncommutative group. The second type of those algebras $(S, +, \cdot)$, called semirings (and sometimes hemirings or halfrings), have in common that, as in the case of rings, both distributive laws are demanded; but $(S, +)$ as well as $(S, \cdot)$ are only assumed to be arbitrary semigroups. They differentiate with respect to miscellaneous further ringlike properties which are included or not by different authors. Some of these properties will be discussed later together with the definition of hemirings and semirings used in the book under consideration. We mention in this context also that a common generalization of nearrings and semirings, called seminearrings, has been investigated in several papers (cf. the references given in the chapters about nearrings and nearfields and about semirings and semifields in Volume 1 of the *Handbook of Algebra*, Elsevier Science Publishers).

Semirings, in the general setting as described above or with more restrictive assumptions, arise naturally in such diverse areas of mathematics as combinatorics, functional analysis, topology, graph theory, Euclidean geometry, ring theory including partially ordered rings, optimization theory, automata theory,