The book by J. S. Golan reviewed here is the first monograph on semirings to appear. It will be very helpful to everyone who works in this field or who needs statements on semirings for any application. In particular, the large number of examples of semirings and applications collected in this book with detailed references, many of them taken from recent works in all the different fields mentioned previously, is a most valuable contribution to make semirings more accessible. The first chapters, dealing with the structure of semirings as defined above, present among other topics the building of new semirings from old ones and the concepts of complemented elements, ideals, factor semirings, and morphisms of semirings and their kernels. In the following chapters Euclidean semirings and additively regular semirings are investigated. Chapters 13–17 are devoted to semimodules over semirings, including free, projective, and injective semimodules; the localization of semimodules; and linear algebra over semirings. Then partially ordered and lattice-ordered semirings are considered. The latter are defined as semirings $(S, +, \cdot)$ which are also lattices $(S, \vee, \wedge)$ such that $a+b = a \vee b$ and $ab \leq a \wedge b$ hold for all $a, b \in S$. Another important tool, in particular for applications, are infinite sums in semirings, defined in an abstract way and subjected to a set of axioms. If each family of elements of a semiring is summable, the semiring is called complete. Chapters 21 and 22 deal with complete lattice-ordered semirings and with fixed points of affine maps $\lambda: H \to M$, where M is a semimodule over a semiring.

In order to put so much material into about 280 pages of text, the presentation of Golan's monograph is concise and requires some effort by the reader. There are also a few minor errors in the mathematics. We mention that in autumn 1993 another book on semirings in German appeared, entitled *Halbringe—algebraische Theorie und Anwendungen in der Informatik* by U. Hebisch and H. J. Weinert. It contains much less material in a more detailed presentation.

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On topological and linear equivalence of certain function spaces, by J. A. Baars and J. A. M. de Groot. CWI Tract 86, Centrum Wisk. Inform., Amsterdam, 1992, 201 pp., DFL 60.00. ISBN 90 6196 411 3

The function spaces dealt with are spaces $\mathrm{C}(X)$ and $\mathrm{C}^*(X)$ of the respectively continuous and bounded continuous real-valued functions on a topological space X.

General topology says that, for every topological space X, there is a Hausdorff completely regular space (= Tychonov space) Y and a continuous surjection $\tau: X \to Y$ such that the mapping $T: \mathrm{C}(Y) \to \mathrm{C}(X)$, $f \mapsto f(\tau)$ is an isomorphism of $\mathrm{C}(Y)$ onto $\mathrm{C}(X)$, whose restriction to $\mathrm{C}^*(Y)$ is also an isomorphism of $\mathrm{C}^*(Y)$ onto $\mathrm{C}^*(X)$ (cf. [6, Theorem 3.9]). Therefore, one can restrict his or her attention to the case when X is a Hausdorff completely regular space without loss of generality. It is clear that the Stone-Čech compactification βX of X plays an essential role in the study of $C^*(X)$ and the real compactification vX of X in the study of $C(X)$.

The spaces $C(X)$ and $C^*(X)$ have a very rich structure; there are many ways to investigate them. Let us mention three of them.

Algebra point of view. The spaces $C(X)$ and $C^*(X)$ are rings. As such one can consider the algebraic properties of $C(X)$ and of its subring $C^*(X)$ and study the characterization of their algebraic properties by means of the topological properties of X. In [6], a basic reference for this point of view, it is said in the introduction that "the major emphasis is placed on the study of ideals, especially maximal ideals, and on their associated residue class rings. Problems of extending continuous functions from a subspace to the entire space arise as a necessary adjunct to this study and are dealt with in considerable detail." This gives a very good idea of what is going on; the review of this book made by J. Dieudonné in the *Mathematical Reviews* (MR22#6994) is "un modèle du genre".

Analysis point of view. The spaces $C(X)$ and $C^*(X)$ can also be considered as vector spaces and be endowed with different Hausdorff locally convex vector topologies. This leads to different areas of research.

The main area is twofold. On one hand, there is the Banach algebra obtained when endowing $C^*(X)$ with the uniform topology; a major reference here is [11] or [15]. On the other hand, there is $C(X)$ endowed with the compact-open topology, denoted by $C_c(X)$. The Gelfand transformation, the Arzela-Ascoli theorem, the Riesz representation theorem giving the link in between the continuous functions and the measures , ... constitute headlines of this very important direction and are so well known that it is not necessary to speak further about this subject here.

Another use of the spaces $C(X)$ started in 1954 when Nachbin [9] and Shirota [16] separately characterized the bornological and the barrelled $C_c(X)$ spaces, providing the first example of a barrelled and nonbornological locally convex space. The family of the $C_c(X)$ spaces was then investigated to get systematically the characterization of the locally convex properties of $C_c(X)$ by means of the topological properties of X. This work was started by Warner in [17]. It has been continued and generalized to more general locally convex topologies by H. Buchwalter, M. De Wilde, K. Noureddine, and others. An account of this can be found in [13]: one can introduce the spaces $C_{\mathscr{P}}(X)$ and $C^*_{\mathscr{P}}(X)$, i.e., $C(X)$ and $C^*(X)$ endowed with Hausdorff locally convex topologies of uniform convergence on the elements of a suitable family $\mathscr{P}$ of bounding subsets of vX and βX, respectively. Cases of particular interest are given by the pointwise topology τ_p and the bounding topology τ_b, giving rise to the spaces $C_p(X)$, $C_b(X)$, and $C^*_p(X)$. Most of the locally convex properties have been characterized, and this was often a way to prove their distinctness. Some properties still are not characterized, such as the Mackey spaces, the Baire spaces, and so on. The extension of this study to the case of continuous functions with values in a locally convex space has also been investigated (cf. [14] for a 1983 survey, but more has been done since then, such as [3]) as well as to the case of weighted subspaces by K.-D. Bierstedt, J. Bonet, R. Meise,

W. Summers, and others; but here no survey is available, so one could start with [3] and trace the story backward.

Topology point of view. The starting point here is a paper of Nagata [10], where one finds the following result: if X and Y are Hausdorff completely regular spaces, then the spaces $C_p(X)$ and $C_p(Y)$ are topologically isomorphic as topological rings if and only if X and Y are homeomorphic. As (cf. [6]) the rings $C(X)$ and $C(Y)$ can be algebraically isomorphic with X and Y nonhomeomorphic, "topologically isomorphic" is essential in that statement. But then one can consider $C_p(X)$ as a mere topological space or as a topological vector space, and two questions arise: (1) When are the spaces $C_p(X)$ and $C_p(Y)$ linearly homeomorphic (resp., homeomorphic)? (2) If they are, for which topological property P is it true that X has property P if and only if Y does? The main results in this direction are rather new (for a large survey, cf. [2]). For example, the answer to question (1) in the case "linearly homeomorphic" is positive for the following properties: pseudocompactness, compactness, σ-compactness [1], and dimension [12]. It is negative for local compactness, first countability, second countability, metrizability, weight, and character. As question (2) is concerned, it receives a positive answer for density [7] and cardinality [2].

The book under review deals with these two questions, extending them also to the topologies τ_c and τ_b. It is partly a survey of the results obtained by the authors, sometimes with the collaboration of J. Pelant and J. van Mill, but contains generalizations of those results as well as new matter. It is largely self-contained (for example, a rather thorough discussion of the ordinal numbers is included).

Chapter 1 is introductory but already contains results such as: if X is a completely metric space, Y is a metric space, and there is a continuous linear surjection from $C_p(X)$ onto $C_p(Y)$, then Y is completely metric. (Functional analysts will not appreciate the choice of the neighbourhoods $\langle f, K, \varepsilon\rangle$ which correspond to no seminorm and to misleading ideas, nor the presentation of the dual of $C_p(X)$ and $C_p^*(X)$. But these are minor things, since the book does not really deal with the locally convex setting.)

In Chapter 2 one finds the isomorphical classification of the spaces $C_p(X)$ when X is a locally convex space of one of the following classes: (1) compact zero-dimensional metric spaces, (2) compact ordinals, (3) σ-compact ordinals, (4) separable metric zero-dimensional spaces. Case (1) was treated for the $C_c(X)$ spaces in [4]; the result is similar, but the method is different. Case (2) was treated for the $C_p(X)$ in [8]; here this result is extended, and the statement for the $C_p(X)$ case is similar, but the method is rather different. In cases (3) and (4) the authors treat the $C_c(X)$ spaces too.

Chapter 3 deals with the nonlocally compact spaces X and homeomorphic spaces of the type $C_p(X)$. Here one finds an example of a compact space X and of a noncompact space Y such that $C_p(X)$ and $C_p(Y)$ are homeomorphic.

Chapter 4 deals with the linearly homeomorphic spaces of the type $C_p(X)$. For instance, there are locally compact countable metric spaces X and Y such that the spaces $C_p(X)$ and $C_p(Y)$ are linearly homeomorphic although the spaces $C_p^*(X)$ and $C_p^*(Y)$ are not.

The book is written in the "definition-statement-proof" way but contains lots of examples, comments, remarks, and references, which will make it pleasant to read by interested people and specialists. It deals considerably with topology and ordinals, much less with locally convex spaces.

(*Note.* Let me point out the paper [5] which just appeared on the same subject.)

References

1. A. V. Arhangelskii, *On linear homeomorphisms of function spaces*, Soviet Math. Dokl. **25** (1982), 852–855.
2. ———, *A survey of C_p-theory*, Questions Answers Gen. Topology **5** (1987), 1–109.
3. K.-D. Bierstedt, J. Bonet, and J. Schmets, (DF)-*spaces of type* $CB(X, E)$ *and* $\overline{CV}(X, E)$, Note Mat. **X** Suppl. 1 (1990), 127–148.
4. C. Bessaga and A. Pelczynski, *Selected topics in infinite-dimensional topology*, Monograf. Mat., vol. 58, PWN, Warszawa, 1975.
5. R. Cauty, T. Dobrowolski, and W. Marciszewski, *A contribution to the topological classification of the spaces* $C_p(X)$, Fund. Math. **142** (1993), 269–301.
6. L. Gilman and M. Jerison, *Rings of continuous functions*, Univ. Ser. Math., Van Nostrand, Princeton, NJ, 1960; 2nd ed., 1976.
7. J. A. Guthrie, *Ascoli theorems and the pseudocharacter of mapping spaces*, Bull. Austral. Math. Soc. **10** (1974), 403–408.
8. S. V. Kislyakov, *Classification of spaces of continuous functions of ordinals*, Siberian Math. J. **16** (1975), 226–231.
9. L. Nachbin, *Topological vector spaces of continuous functions*, Proc. Nat. Acad. Sci. U.S.A. **40** (1954), 471–474.
10. J. Nagata, *On lattices of functions on topological spaces and functions on uniform spaces*, Osaka Math. J. **1** (1949), 166–181.
11. M. A. Naimark, *Normed rings*, Noordhoff, Groningen, 1959.
12. V. G. Pestov, *The coincidence of the dimension* dim *of l-equivalent topological spaces*, Soviet Math. Dokl. **26** (1982), 380–383.
13. J. Schmets, *Espaces de fonctions continues*, Lecture Notes in Math., vol. 519, Springer, Berlin, 1976.
14. ———, *Spaces of vector-valued continuous functions*, Lecture Notes in Math., vol. 1003, Springer, Berlin, 1983.
15. Z. Semadeni, *Banach spaces of continuous functions*, Monograf. Mat., vol. 55, PWN, Warszawa, 1971.
16. T. Shirota, *On locally convex vector spaces of continuous functions*, Proc. Japan Acad. **30** (1954), 294–298.
17. S. Warner, *The topology of compact convergence on continuous function spaces*, Duke Math. J. **25** (1958), 265–282.

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