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Smoothness and renormings in Banach spaces, by Robert DeVille, Gilles Godefroy and Vaclav Zizler. Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 64, Longman Scientific & Technical, Harlow, 1993, xii + 220 pp., $130.00. ISBN 0-582-07250-6

Renorming theorems has been the key to applying Banach space ideas to other areas of mathematics almost from the beginning of the theory. In the work of Clarkson, Dunford, and Morse we find simple renormings clearing the way to differentiability results. Krein, Krasnoselskii, and Milman used renormings to derive basic properties about openings of subspaces in their work on defect numbers of operators, results rediscovered and given substantially more complicated proofs (unaided by renormings) by Tikhomirov in his study of widths of sets and best approximants. Kadets renormed separable spaces in a locally uniformly convex manner and, using this, introduced good coordinate systems in separable spaces on his way to proving all separable infinite-dimensional Banach spaces are homeomorphic. Bonic and Frampton showed c_0 has a C^∞ smooth norm (away from 0) and used this, along with companion results for other classical spaces, to carry out their work in Banach manifolds. Pisier tied vector-valued versions of the Burkholder-Davis-Gundy inequalities for martingale difference sequences to the vector space's superreflexivity; probability gained as did Banach space theory.

Troyanski renormed spaces with fundamental weakly compact sets in a locally uniformly convex fashion and in so doing showed that weakly compact convex sets in Banach spaces have lots of strongly exposed points; soon this was to be understood as basic to the study and applications of Banach space-valued martingales. Dineen and Timoney renormed spaces with isomorphic copies of c_0 inside them to attain isometric copies and used this to classify bounded domains in complex Banach spaces that are biholomorphic with finite products of irreducible Banach analytic manifolds.

Renormings often ease the way to a proof and can be the basis of substantial new connections between the theory of linear spaces and other areas of mathematical endeavor.

Until the book under review, most texts on Banach space theory spent only a chapter or two (at most) on renormings and then usually with other purposes in mind. Very rarely was the topic of smoothness (the other key word in the book's title) dealt with in any but first-order terms. This lack of coverage no longer exists.

The authors have set out to expose many of the basic ideas and applications of smoothness and renormings in Banach spaces. They have succeeded admirably. The result is a book chock-full of delicious tidbits packaged so as to have maximal impact. The seven chapters are remarkably independent and, except for the last chapter, could each be the basis for a semester-long seminar for students who have had a year of functional analysis. There is a sincere effort to explain what is really going on, and this effort has resulted in a level of exposition rarely encountered by this reviewer.

Each chapter is followed by a brief but very informative set of notes and remarks and a section on open problems. The authors are not encyclopedists and are completely unafraid to send the reader to the literature for proofs too complicated to shed any light on their discussions; for example, they do a beautiful job on the separable version of James's norm attaining functionals theorem but refer to the master himself for the nonseparable case. They are also not afraid of presenting complicated proofs that are particularly germane to their discussions; witness, for instance, their development of higher-order smoothness, the first place much of this material has been carefully detailed anywhere.

While expository in nature, the book contains many new results that play an important role in the development of the theory. Old results are often given new insightful proofs usually much shorter than other, existing proofs. Always, though, sympathy for the reader is plainly present.

The only drawback of this beautiful text is its price, $130. I fear it will result in substantial photocopying fees around the world. Too bad. This book is well worth the price.

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