volume makes a nice companion work to his excellent 13 *Lectures on Fermat's Last Theorem* (Springer, New York, 1979).

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Non-classical elastic solids, by M. Ciarletta and D. Iesan. Pitman Research Notes in Mathematics, vol. 293, Longman Scientific & Technical, Harlow, Essex, 345 pp., $74.95. ISBN 0-582-22716-X

Let X_A denote a point in a body, and suppose the body deforms and the point moves to x_i. The deformation gradient F_{iA} is defined by

$$F_{iA} = \frac{\partial x_i}{\partial X_A} = x_{i,A}.$$

A classical elastic solid is one for which the stress tensor σ_{rs} and the internal energy U depend on $x_{i,A}$ and possibly X_A, i.e.,

$$\sigma_{rs} = \sigma_{rs}(x_{i,A}, X_A),$$
$$U = U(x_{i,A}, X_A).$$

In the linear approximation this gives σ_{rs} as a linear function of strain $x_{i,A}$, which is Hooke's law.

Over the past 30 years or so there has been significant interest in elastic-like materials which cannot be adequately described by the classical theory of elasticity. The present book is concerned with mathematical aspects of three theories which depart from classical elasticity theory.

A beautiful exposition of two of the nonclassical elastic solid theories may be found in Truesdell and Noll [6, p. 389]. They point out that Cauchy's second law in Continuum Mechanics is a constitutive assumption which says there are neither body couples nor couple stresses. A class of nonclassical materials are those for which there may be couple stresses or body couples present, and these are called polar materials; this theory was first developed by E. and F. Cosserat in 1907. In fact, it was Duhem who suggested including effects of direction via sets of points with vectors attached to them, thus giving rise to the theory of oriented media. This theory was developed by the Cosserats. Another generalization of classical elasticity is to elastic materials of grade 2 or higher, and this is also lucidly explained by Truesdell and Noll [6].

The theory of oriented media leads naturally to a theory of elastic rods (Antman [1]), or to elastic shell theory (Naghdi [3]). Also, it offers a very successful way to describe liquid crystals, a class of materials surely known to almost everyone in the developed world. Inclusion of body couples arises naturally in the industrially important field of ferrohydrodynamics (Rosensweig [5]). Here, the ferrofluid is a suspension of magnetic particles in a carrier liquid and the resulting fluid possesses giant magnetic response. Since the magnetic particles can spin on their own, the idea of a body couples fits naturally. Another success of oriented media is to the description of turbulence; the papers of Marshall and Naghdi [2] illustrate this beautifully.

The book by Ciarletta and Iesan concentrates on three generalizations of the classical theory of elasticity, namely, that of nonsimple elastic bodies (Chapters 1–3), elastic solids with microstructure (Chapters 4–6), and elastic materials which contain voids (Chapters 7, 8).

In the class of nonsimple elastic bodies they restrict attention to elastic materials of grade 2 for which the stress and internal energy contain also the derivative of the deformation tensor, i.e.,

$$\sigma_{rs} = \sigma_{rs}(x_{i,A}, x_{i,AB}, X_A),$$
$$U = U(x_{i,A}, x_{i,AB}, X_A).$$

By elastic solids with microstructure they study bodies where the stress and internal energy depend also on a variable x_{iA} called a microscopic deformation, or sometimes a dipolar displacement, and the derivative of x_{iA}. Thus, for this class of materials

$$\sigma_{rs} = \sigma_{rs}(x_{i,A}, x_{iB}, x_{iB,K}, X_M),$$
$$U = U(x_{i,A}, x_{iB}, x_{iB,K}, X_M).$$

In particular the above class may be seen to contain Cosserat materials as a special case by taking $x_{ij} = \epsilon_{ijk}\phi_k$, where ϕ_k is a microrotation field. The theory of elastic materials with voids was developed by Nunziato and Cowin [4] and is physically important as many rubber-like materials do contain air-filled pores. For this theory there is the usual deformation

$$x_i = x_i(X_A, t),$$

but there is also a volume fraction field

$$\nu = \nu(X_A, t), \qquad 0 < \nu \leq 1,$$

such that the mass density satisfies

$$\rho = \nu\gamma,$$

where γ is the density of the elastic matrix material. The internal energy for such a material has constitutive relation

$$U = U(x_{i,A}, \nu, \nu_{,B}, \dot{\nu}, T, X_M),$$

where $\dot{\nu} = d\nu/dt$ and T is the temperature.

The mathematical content of the book is to review several types of variational theorems, reciprocal theorems, existence theorems, continuous dependence results, uniqueness results, results of St. Venant type concerning the rate of decay of displacement in a cylinder, and analysis of plane waves and shock waves. In the treatment of elastic bodies with voids in addition to results of the above type the writers treat the important topic of acceleration waves. An acceleration wave is a propagating singular surface across which the acceleration may suffer a finite jump discontinuity. The evolutionary behaviour of such a wave is resolved. If the wave amplitude becomes infinite, this can lead to shock wave formation.

Overall the book is clearly written and will certainly be a useful reference to anyone working in the field. There are many references, and some of those are in Eastern bloc journals which may not be so familiar in the West. The book is not produced in TEX but is nevertheless produced by a pleasant-to-read word-processing system.

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Hilbert's Tenth Problem, by Yuri V. Matiyasevich. The MIT Press, Cambridge, MA, and London, 1993, xxii + 264 pp., $45.00. ISBN 0-262-13295-8

It was the reading of a translation of Diophantus's books on arithmetic that led Pierre de Fermat to found modern number theory and the study of what are now called Diophantine equations. Diophantine equations are nothing more than equations between polynomials in several variables, their *Diophantineness* lying not in the nature of the equations but in that of the solutions being sought. Diophantus and algebraic geometers like rational solutions, while Fermat and his successors prefer integral solutions. Fermat himself is associated with two important Diophantine equations, namely: the Fermat equation,

$$x^n + y^n = z^n,$$

for which he claimed to have only the obvious solutions for $n > 2$; and the so-called Pell equation,

$$x^2 - Dy^2 = 1, \qquad D \text{ not a perfect square},$$

the name of which originates in an error of attribution on Leonhard Euler's part.