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Hopf algebras and their actions on rings, by S. Montgomery. CBMS Regional Conference Series in Mathematics, vol. 82, American Mathematical Society, Providence, RI, 1993, xiv + 238 pp., $32.00. ISBN 0-8218-0738-2

This book is a good guidebook to recent developments in Hopf algebra theory with an emphasis on their actions and coactions on algebras. It contains some classical results which overlap with the foregoing textbooks [S] and [A] but includes new applications and treatments. Most of the results have not previously appeared in book form. They are described in a self-contained way with or without proofs; and when the proofs are omitted, appropriate references to the literature are specified. This book could also be used as a textbook for graduate students.

Kaplansky [K] proposed ten conjectures on Hopf algebras in 1975. Some of them have recently been solved, including a kind of Lagrange's theorem for Hopf algebras settled by Nichols and Zoeller in 1989, which tells that a finite-dimensional Hopf algebra is free over any Hopf subalgebra. Chapter 3 is devoted to its proof, applications, and related topics. The authors chose a good time to publish such a basic theorem in book form, for it is useful and sometimes indispensable in the subsequent discussion of finite-dimensional Hopf algebras. Other contributions to Kaplansky's conjectures are mentioned without proof (Theorems 2.5.2, 2.5.3 and Conjecture 3.2.2).

The author, S. Montgomery, has contributed much to group actions on rings [M] and their generalizations to Hopf algebras (especially finite dimensional). One purpose of this monograph seems to develop the author's and her collaborators' results systematically, including prerequisites, backgrounds, and other related topics.

There have been many studies on group actions on rings and their relations with Galois theory [P]. When a Hopf algebra H acts on an algebra A, one can form the subalgebra of invariants A^H and the smash product $A\#H$, a generalization of the skew group ring, which is further generalized to the crossed product $A\#_\sigma H$ involving a two-cocycle σ. Ring-theoretic studies of the algebras A^H, $A\#H$, and $A\#_\sigma H$ as well as relations with Hopf Galois theory are

a central theme of this book. One advantage of such a treatment is that one can deal with group actions and coactions in the same framework. (By a group coaction, I mean a group-graded ring.) One finds such an application in §9.5.

Actions of finite-dimensional Hopf algebras are discussed in Chapter 4 and §8.3. Although appearing far apart, one finds an interesting connection between Morita and Galois theories (Theorems 4.5.3 and 8.3.3). The author proposes various questions supplied with the best answers known to date.

Crossed products $A\#_\sigma H$ are studied in detail in Chapter 7. This notion was introduced by Doi and the reviewer during their study of Hopf Galois extensions with the normal basis property which are also called *cleft* extensions (Theorem 7.2.2 and Corollary 8.2.5). The same construction was given by the author and her collaborators independently. Various conditions are presented so that $A\#_\sigma H$ is semisimple, semiprime, or semiprimitive in §7.4.

The generalization of usual group Galois theory to Hopf algebras is called *Hopf Galois theory* and is discussed in Chapter 8, with an emphasis on the normal basis property which is closely related to crossed products. When $A \subset B$ is an H-Galois extension with a Hopf algebra H, one may consider the Galois correspondence

$$\overline{H} \mapsto B^{\mathrm{co}\,\overline{H}}$$

from the set of quotient Hopf algebras $\overline{H}$ to the set of subalgebras of B containing A. In particular, one may ask when $B^{\mathrm{co}\,\overline{H}} \subset B$ becomes a $\overline{H}$-Galois extension. A special case of this question restricted to extensions with the normal basis property is asked (Question 8.4.1), and Schneider's answers are presented (Proposition 8.4.4 and Theorem 8.4.6). In fact, he has contributed to the original question itself.

Chapter 6 contains Koppinen's version of the Skolem-Noether theorem for Hopf algebras (Theorem 6.2.3), Masuoka's result on maximal inner subcoalgebras (Theorem 6.3.1), and the author's new result on extending the Hopf algebra action to some quotient algebra (Proposition 6.4.5 and Theorem 6.4.6).

The duality for actions discussed in Chapter 9 has its roots in operator algebra. The Blattner-Montgomery duality theorem tells that if one iterates the procedure of making the smash product twice, then one gets some tensor product of algebras. Its complete formulation and self-contained proof are given (Theorem 9.4.9) as well as its applications.

This book ends with a brief introduction to *quantum group theory*, one of the most exciting topics in mathematics today. The basic examples of Hopf algebras appearing in this theory—such as $U_q(\mathbf{g})$ for $\mathbf{g}$, a semisimple Lie algebra, or $\mathscr{O}_q(G)$ for G, an affine algebraic group—are noncommutative, noncocommutative but are almost cocommutative or almost commutative. Its precise meaning is clarified in Chapter 10, first from a purely Hopf algebraic viewpoint (§§10.1 and 10.2) and second from a categorical viewpoint (§10.4). For a finite-dimensional Hopf algebra H, Drinfeld has defined some unusual Hopf algebra structure on $H^* \otimes H$, making the so-called *Drinfeld double*. Its construction and basic properties are reviewed after the treatment of Majid and Radford (§10.3) as well as its categorical meaning (Proposition 10.6.16). (See [DT] for another approach.)

Quantum Hopf algebras provide many good natural examples to illustrate abstract Hopf algebra theory. In fact, the author mentions such examples through-

out the book. I will close by adding one more example which I found just as I was asked to review this book. During his study of the quantum algebra at a root of 1, Lusztig [L] defines some *normal* finite-dimensional Hopf subalgebra u and shows that the quotient Hopf algebra by u is identified with the hyperalgebra of a simply-connected semisimple algebraic group G. In fact, this extension of Hopf algebras is *cleft* (Theorem 7.2.2) so that the quantum algebra at a root of 1 (of odd order) has the form of a crossed product $u \#_\sigma hy(G)$. This fact has some applications which I will talk about elsewhere.

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The computational complexity of differential and integral equations, by Arthur G. Werschulz. Oxford University Press, 1991, ix + 331 pp., $55.00. ISBN 0-19-853589-9

Computational mathematics is generally thought to consist of the following subfields:

(1) numerical solution of differential and integral equations,
(2) numerical linear algebra,
(3) computational geometry, and
(4) discrete algorithms and complexity theory.

There has been significant interactions between most of these areas with one notable exception, namely, the first and the last. The most complete overview of all four subject areas has been in the highly influential and brilliantly lucid book on applied mathematics by Gilbert Strang [1]. Even there the full power of complexity theory did not emerge until the seventh chapter, where discrete algorithms were considered. In earlier chapters where differential equations were involved, complexity theory took the form of simple operation counts.

In the book under review, Werschulz makes an attempt to go well beyond this and apply the full force of complexity theory to algorithms designed for the