The subject is flourishing. The books under review will make it much easier for newcomers (and oldtimers) to participate in future developments.

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PETER ROSENTHAL
UNIVERSITY OF TORONTO
E-mail address: `rosent@math.toronto.edu`

BULLETIN (New Series) OF THE
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Harmonic maps and minimal immersions with symmetries, by James Eells and Andrea Ratto. Annals of Mathematics Studies, vol. 130, Princeton University Press, Princeton, NJ, 1993, 228 pp., $19.95. ISBN 0-691-10249-X.

A harmonic map between Riemannian manifolds generalizes the notion of a harmonic function: if the target manifold is viewed as embedded in some Euclidean space, then a map $\varphi: M \to N \subset \mathbb{R}^k$ is harmonic if the Laplacian of φ (as a map into $\mathbb{R}^k$) is perpendicular to N. This is precisely the Euler-

Lagrange equation for the Dirichlet energy subject to the constraint of taking values in N. There is motivation from physics and geometry for studying such maps: harmonic maps of Riemann surfaces into complex projective spaces and Grassmannians (nonlinear sigma models) or Lie groups (chiral models) share a number of properties of Yang-Mills theory: gauge theoretic formulation as Yang-Mills-Higgs fields, conformal invariance, and instanton solutions. The connection with geometry is given by the fact that every minimal immersion is harmonic and that the Gauss normal map of a submanifold in Euclidean space is harmonic precisely when the submanifold has parallel mean curvature. Suitable variations of the latter observation make the study of constant Gauss curvature surfaces, constant mean curvature surfaces, and Willmore surfaces in 3-space a special case of the study of harmonic surfaces in homogeneous spaces.

If the target space is nonpositively curved, existence of harmonic maps in a given homotopy class is obtained by deforming an initial map along the heat flow. For positively curved target manifolds—which include many of the cases of interest in physics and surface geometry—only local existence results are known. Thus, to obtain nontrivial examples and global existence, different approaches have been sought. For 2-dimensional domains considerable progress has been made in describing harmonic maps into homogeneous target spaces (typically positively curved): every harmonic map of the 2-sphere into complex projective space is obtained as an element of the Frenet frame of a rational curve, a classical result due to Eells and Wood. Similar reductions to holomorphic data via twistorial constructions hold in general for symmetric target spaces and Lie groups, in which case one recovers the harmonic map from an appropriate holomorphic lift into a finite-dimensional stratum of the loop group. For higher genus domains the loop group approach has proven to be successful recently: harmonic maps of 2-tori into symmetric spaces and Lie groups are obtained from specific finite-dimensional integrable systems on loop algebras, and harmonic maps of genus ≥ 2 surfaces are obtained by integration of holomorphic loop algebra-valued 1-forms.

If the domain is higher dimensional, one cannot (except for the pluriharmonic case) expect constructions of the above type. It thus seems natural to impose symmetry conditions to obtain existence results and explicit examples. This leads to the study of harmonic maps which are equivariant with respect to a group action by isometries on domain and target. If the group action on the domain has 1-dimensional quotient, the harmonic map equation reduces to a second-order nonlinear ODE for a curve in the quotient of the target space. Typically one seeks global solutions with singular boundary conditions or prescribed asymptotic behaviour. In the case of minimal submanifolds the ODE on the quotient is the geodesic equation after the natural quotient metric on the target has been scaled by the orbit volume function. This idea has been used to describe the following: all S^1-invariant minimal 2-tori in S^n and S^1-invariant Willmore tori in S^3 (where the equations turn out to be the classically completely integrable Neumann system), rotational hypersurfaces in S^n, and nontrivial embedded rotational hyperspheres of constant mean curvature in R^n, $n \geq 4$ (showing that Hopf's Theorem on constant mean curvature spheres fails in higher dimensions). In each of these cases a careful qualitative study of the corresponding ODE has to be carried out, which brings us to the main theme of the present book. It begins with an easy-to-read overview of harmonic map theory and its applications to geometry; this includes concise outlines of classical results—Hopf's and Alexandrov's theorems on constant mean curvature spheres and the Ruh-Vilms Theorem—and of Wente's construction of a constant mean curvature torus (following an approach by Abresch). The authors then discuss the equivariant setup. The cases where the quotient of the domain space by the group action is zero- or 1-dimensional are treated in detail. The former leads to the result that any compact homogeneous space is harmonically (and minimally) immersible into some Euclidean sphere. The latter case yields the construction (by studying the corresponding ODE's) of special $SO(n-1)$-invariant minimal hypersurfaces in S^n, of a countable family of embedded $SO(2) \times SO(2)$-invariant minimal 3-spheres in S^4, and of a countable family of embedded $SO(p) \times SO(q)$-invariant constant mean curvature hyperspheres in $\mathbb{R}^{p+q}$, $p, q \geq 2$. The remaining third of the book deals with the construction of harmonic maps from spheres into spheres. The basic idea to obtain such maps is by "joining" eigenmaps. The harmonicity equation reduces to a pendulum-type equation (with variable damping and gravity) for the "joining" function, whose qualitative global behaviour can be studied.

Except for this last part, which is mostly a topic for the expert reader, the book is written for a general audience with some interest in harmonic maps. Each chapter concludes with a welter of notes and comments, giving the reader a more global context as well as many hints for further studies. The pace of the book is comfortable, even when it comes to technical lemmas, which are kept balanced by qualitative statements. To avoid losing focus, the reader is provided with a detailed roadmap at the beginning of the book and of each chapter. The authors conclude with a rather comprehensive reference list. Parts of the book can certainly form the basis for a topic course, and it could be suggested reading for graduate students and researchers with an interest in harmonic maps.

Franz Pedit
University of Massachusetts
E-mail address: `franz@gang.umass.edu`

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Invariant distances and metrics in complex analysis, by Marek Jarnicki and Peter Pflug. Expositions in Mathematics, vol. 9, Walter de Gruyter, Berlin, 1993, xi+408 pp., $98.95. ISBN 3-11-013251-6

The subject of invariant metrics in complex analysis started with Poincaré's investigations of Fuchsian groups, that is, discrete subgroups in the group of all holomorphic automorphisms of the unit disc $E \subset \mathbb{C}$. Poincaré made two crucial observations: first, if one defines the length of a smooth curve $\Gamma \subset E$ by

$$\int_\Gamma \frac{|dz|}{1-|z|^2}$$