

BOOK REVIEW

APPEARED IN BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 32, Number 4, October 1995, Pages 457-458
©1995 American Mathematical Society
0273-0979/95 \$1.00 + \$.25 per page

Green functions for second order parabolic integro-differential problems, by
M. G. Garroni and J. L. Menaldi. Longman Scientific & Technical, Harlow,
Essex, England, 1992, 417 pp., \$57.00. ISBN 3-582-02156-1

The topic of this book is the deep and highly involved treatment of parabolic equations of second order containing a nonlocal term of a special structure. This treatment is general in the sense that all types of boundary conditions are considered with equal weight, both in Hölder- and L_p -spaces.

The equation is $Lu - I(u) = f$, where

$$Lu = \frac{\partial u}{\partial t} - a_{ij}(x, t) \frac{\partial^2 u}{\partial x_i \partial x_j} - a_i(x, t) \frac{\partial u}{\partial x_i} - a_0(x, t)u$$

is a classical linear uniformly parabolic operator of second order with bounded $(\alpha, \frac{\alpha}{2})$ -Hölder continuous coefficients. Chapter 1 collects all the standard material (Schauder estimates in $C_{\alpha, \alpha/2}$ and L_p , maximum principles, etc.) concerning L . The nonlocal operator $I(u)$, modelling jumps in the diffusion process, has (roughly) the structure

$$I(u) = \int_F [u(x + j(x, t, \xi), t) - u(x, t) - j(x, t, \xi) \cdot \nabla_x u(x, t)] m(x, t, \xi) \pi(d\xi)$$

with some σ -finite measure π on F , and $T_\theta(x) = x + \theta j(x, t, \xi)$ is a diffeomorphism for all $\theta \in [0, 1]$, t, ξ . The properties of I are discussed in Chapter 2; especially conditions are given such that $\|I(u)\|_\chi \leq \varepsilon \|D^2 u\|_\chi + c_\varepsilon$ l.o.t. for $\chi = C_{\alpha, \alpha/2}$ or L_p . Due to the “minus” sign in front of $I(u)$ the maximum principle can also be saved; hence this would be enough for just solving the equation via fixpoint arguments.

But the aim of the authors is a different one, as indicated in the title of the book. Hence its middle part (Chapter 3–Chapter 6) gives a dense presentation, construction and discussion of the properties of the fundamental solution and of the Green functions for L under various boundary conditions. One can find the whole classical material, including simple and double-layer potentials, jump relations and Levi’s Parametrix method. The other possibility for constructing the Green function, using the existence theory for the initial boundary value problem for $Lu = f$, is indicated in the third part of Chapter 6.

The last three chapters are devoted to the construction and discussion of the Green functions G for $L - I$. The key tool for the construction of G as a series of terms, each of which solves a Volterra equation, is the definition and study of

a certain scale of Banach-spaces containing these kernel functions. This scale is defined by means of fifteen(!) (semi-)norms. Chapter 8 contains the construction of G , and properties and estimates of G are discussed in Chapter 9.

Due to the subject, the contents of the book are highly technical—a research note in the best sense. It has the further merit of containing the classical material for linear parabolic equations with variable coefficients for all types of boundary values in a form which allows a citation.

MICHAEL WIEGNER
INSTITUT FÜR MATHEMATIK
UNIVERSITÄT BAYREUTH
E-mail address: michael.wiegner@uni-bayreuth.de