Partial differential equations I, II, III, by Michael Taylor, Springer - Verlag, New
York; Vol. I *Basic Theory*, 1996, xxi+563 pp., $69.00. ISBN 0-387-94653-5;
Vol. II *Qualitative studies of linear equations*, 1996, xxi+528 pp., $69.00. ISBN
0-387-94651-9; Vol. III *Nonlinear equations*, 1996, xxi+610 pp., $69.00. ISBN
0-387-94652-7

It is hard to find a field of mathematics as broad as partial differential equations.
The following is an incomplete list of active topics in partial differential equations
in the last twenty years:

- Pseudodifferential operators.
- Microlocal analysis and propagation of singularities.
- Fourier integral operators, caustics, grazing and gliding rays.
- Spectral and inverse spectral problems.
- KdV equation and other completely integrable equations.
- Index theory of elliptic operators.
- Variational inequalities and obstacle problems.
- Nonelliptic boundary value problems and hypoelliptic equations.
- Shock waves and quasilinear hyperbolic equations.
- Regularity problems for elliptic and parabolic equations.
- Homogenization in elliptic and parabolic equations.
- Euler's and Navier-Stokes equations.
- Problems of differential geometric origin: Yamabe problem, harmonic maps, minimal surfaces, Monge-Ampere equation, etc.
- Yang-Mills equations.
- Scattering theory including theory of resonances, N-body scattering and inverse scattering problems.
- Manifolds with singularities, corners, wedges , etc.
- Nonlinear wave equations and Einstein's equations of general relativity.
- Mathematical problems of quantum and statistical mechanics.
- Vanishing viscosity solutions of nonlinear equations.
- Compensated compactness, defect measures, concentrated compactness principle.

It is only natural that even people working in partial differential equations know
very little about some areas. There are many books on various subjects of par-
tial differential equations. Most of them are focused either on linear or nonlinear
equations. These three volumes are a rare attempt to combine the main topics of
partial differential equations both linear and nonlinear in one book.

Michael Taylor is one of the few people who would be able to do this. He not only
knows a wide range of subjects, but he has actually worked on most of the topics
of this book. This has resulted in a high level of exposition maintained throughout
the whole book.

The book consists of three volumes: the first volume contains introductory linear
partial differential equations, the second volume treats more advanced topics of

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linear equations, and the third volume is on nonlinear pde. There are also lengthy appendices on differential geometry and functional analysis that make the book completely self-contained.

There is no other book where such different subjects as brownian motion and pseudodifferential operators, quasilinear hyperbolic equations and index theory, $\bar{\partial}$-Neumann problem and Einstein's gravitational equations, nonlinear elliptic equations and caustics coexist under the same roof. Despite its breadth most of the topics are treated on a more advanced level than one could expect of such a broad book. For example, the book includes with proofs

- The viscosity method and Di Perna results for 2×2 systems of conservation laws,
- Einstein's gravitational equations,
- Di Georgi - Nash - Moser estimates and Krylov - Safonov estimates for nonlinear elliptic and parabolic equations,
- Young measures,
- Paradifferential operators,
- Calderon - Zygmund theory,
- Proof of the index formula using Getzler's method,
- Topics in differential geometry such as prescribed Gauss curvature, harmonic maps and minimal surfaces,
- The Nash theorem on isometric imbedding of Riemannian manifolds.

In addition to the subjects mentioned above the second and third volumes also cover the Weyl calculus of pseudodifferential operators, the parametrix for elliptic boundary value problems and for the heat equation, microlocal regularity, heat asymptotics and spectral asymptotics for the Laplacian on manifolds, the Laplace operator on the cone, Dirac operators and the Chern - Gauss - Bonnet theorem, subelliptic estimates, the $\bar{\partial}$ - Neumann problem for strictly pseudoconvex domains, the Bergman projection and Toeplitz operators, direct methods in calculus of variations, Gagliardo - Nirenberg - Moser estimates, Monge - Ampere equations, reaction - diffusion equations, the Stefan problem, the Euler and Navier - Stokes equations, coupled Maxwell - Einstein equations, Weiner measures and the Feynman - Kac formula, martingales, Newton capacity, stochastic integrals and stochastic differential equations and others.

The first volume can easily serve as a textbook for a first-year graduate course in partial differential equations. It contains the theory of the first order scalar nonlinear pde, the Laplace operator on a Riemannian manifold, Sobolev spaces, the Fourier transform and theory of distributions, elliptic boundary value problems, hyperbolic systems, energy estimates, finite propagation speed and geometric optics. In addition there are more advanced topics, such as the proof of the Riemann mapping theorem for domains with rough boundaries, the formation of caustics, the Hodge decomposition theorem and harmonic forms, Lorentz manifolds, and Maxwell equations. There are large supporting sections on manifolds, vector bundles, differential geometry and linear operators.

The second and third volumes will be very useful to mathematicians who want to study in depth advanced topics of partial differential equations, relieving them

from the need to search through the journal articles.

It is a very good book. Everyone interested in pde should read it.

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