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Classical topics in complex function theory, by Reinhold Remmert (translated by Leslie Kay), Springer-Verlag, xix+349 pp., $49.95, ISBN 0-387-98221-3

The author has selected several topics in one variable function theory and done an excellent job of presenting them. The topics surely represent the author's interests and reflect in his opinion major landmarks on the way towards the development of this mature subject. His approach has been consistent throughout. Having chosen a topic, he gives us the most enlightening proofs of the material under study. He includes in these sections historical material pertaining to the contributions of various mathematicians. In doing this the author leads us through the thought process of the individuals involved as well as the influence of schools of mathematics leading up to the proofs. For example in the treatment of the Runge theory he discusses the path that Runge followed to produce a series of consecutively stronger results leading to the full strength of the theory. He includes the statement that certain significant counterexamples of Runge were not noted at the time. He indicates a certain gap in the historical knowledge of the subject to the effect that he does not have a proper attribution as to who first produced a sequence of holomorphic functions converging pointwise to a discontinuous limit function. This is delightful to read and will be appreciated by the novice as well as the expert. The inclusion of the original document by Riemann, page 182, will probably not be appreciated by many of us. The book is replete with diagrams and pictures. These are very well chosen and illustrate the points needed. They add immensely to the pedagogical presentation.

The topics chosen for study represent some of my favorites. For those not so familiar with classical function theory, as well as the experts, the choice of a specific section in the book offers a unique encapsulated approach to one important area of study in the field. For example the choice Domains of Holomorphy leads to the beginning definition of what is a domain of holomorphy and how it contrasts with a maximal domain of existence. At this point the main Existence Theorem (For every domain G in C , there exists a function f holomorphic in G such that G is the domain of holomorphy for f) is stated. He supplies two interesting proofs of this result. This is followed by information on the history of the concept and in turn this is followed by a glimpse of several variables.

In this latter material he includes a brief view of mathematical developments that have followed since these original theorems were proven. In many cases these future developments point in the direction of the theory of several complex variables. This subject has already been the focus of intense investigations. Important results indicating striking differences and important similarities are already in standard book form. Although he refers to some of this text material (i.e. the reference to [Ra] on page 202 and to [Ho] on page 294 of his text), I think the inclusion of the texts by [Kr] and [Sh] would also be pertinent. The latter text has had a strong influence on the education of students in the Soviet school.

So we have a book written by an erudite author, an outstanding mathematician in his own field who has a gift for writing. The style is clear, terse and complete.

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The choice of material covers some of the most profound developments in the field. I would have included some others (*i.e.* Carleson measures) with more current influential impact on analysis and omitted a few (Chapter 11 and perhaps 14) that are covered in detail but are more well known and that are not so influential on current modern classical (complex) analysis. The highlights of the text include a very thorough and general treatment of zeros of holomorphic functions, domains of holomorphy, mapping theory, the Bloch, Picard and Schottky Theorems and a comprehensive treatment of the Runge theory.

I did not compare the translated text to the original in German but note that the translator, Dr. Leslie Kay, is also a specialist in this area. It would be helpful in reading this text to have his first volume [Re] in hand as he does refer to it in several places. There is one important phrase that will confuse the non-expert in analysis. On page 180, part (ii) of the equivalence theorem (the equivalences in Riemann mapping theorem) it is stated that every function holomorphic in G is integrable in G. This has a well accepted meaning in mathematical English, and it is not what is written. The difference in being an integral (or primitive) as opposed to being integrable is clear, and it is the former that we want. Incidentally the same error occurs in [Re] on page 292, Theorem (ii).

In the monetary aspect of the publishing industry when the publishing company contemplates printing and publishing a book, they ask the question Who will buy this book? I want to ask and answer a slightly different version of this question. What professional purpose does this text serve to our current mathematical community? The material is for the most part classical, dating in some instances to the time period from the 1800s to the mid 1930s. Many texts are written by distinguished authors covering parts of this material in many languages. What does this one do? It presents the material in a masterly way! Although the material is older the author has put these deep results in as modern a setting as possible. In particular there are several places (*i.e.* pages 107-112, Iss'sa's Theorem) where an algebraic point of view pervades. The service of tracing the important aspects of the history of a significant branch of our field of study is a good contribution, especially when it comes from such an expert in the field. He is viewing the subject from the great vantage point of time. Finally, because of his accomplishments he has a distinguished position from which to view it. I suggest this book as an excellent source for classical material in this area. I think if you pick it up and study just one section you will agree with my assessment.

References

[Kr] S.G. Krantz, Function Theory of Several Complex Variables, second edition, Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, CA, p. 557 (1992). MR **93c:**32001

[Re] R. Remmert, Theory of Complex Functions (translation by R.B. Burckel), Springer Verlag, New York, NY, p. 453 (1991). MR **91m:**30001

[Sh] B. V. Shabat, Introduction to Complex Analysis, Part II, Functions of Several Variables, Translations of Mathematical Monographs Vol.110, AMS, Providence, RI (1992). MR **93g:**32001

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