BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 35, Number 4, October 1998, Pages 341–342
S 0273-0979(98)00760-5

Combinatorial geometry, by János Pach and Pankaj K. Agarwal, Wiley-Interscience, New York, 1995, xiii+354 pp., $67.00, ISBN 0-471-58890-3

In the last twenty years we have witnessed a tremendous flow of outstanding results in the fields of combinatorics, convex geometry and discrete geometry. The works of Paul Erdős, László Fejes Tóth and C. Ambrose Rogers initiated new approaches to several classical problems of mathematics and made connections to several areas of pure and applied mathematics. For the schools that they founded the recent development of computer technology presented a new source of inspiration. As a result there was a tremendous need for an advanced undergraduate-level book that connects and extends the classical undergraduate-level geometry to the modern and very applicable fields of combinatorial, convex and discrete geometry. The book of János Pach and Pankaj K. Agarwal does exactly this job in a very elegant way, introducing several of the most important notions of combinatorial geometry and explaining a great number of outstanding results of the field on somewhat less than 300 pages. The clear style of the book and the numerous exercises in the sixteen chapters make the book ideal either for a two-semester senior-level undergraduate course or for a one-semester junior-level graduate course on combinatorial geometry. The following brief summary of the sections shows the elegant choice of the selected topics as well as the self-contained nature of the book.

Part I of the book is devoted to the theory of packing and covering, mainly influenced by the works of László Fejes Tóth and C. Ambrose Rogers. The eight sections on (1) Geometry of Numbers, (2) Approximation of a Convex Set by Polygons, (3) Packing and Covering with Congruent Convex Discs, (4) Lattice Packing and Lattice Covering, (5) The Method of Cell Decomposition, (6) Methods of Blichfeldt and Rogers, (7) Efficient Random Arrangements and (8) Circle Packings and Planar Graphs describe some of the celebrated classical results and some of the recent results of the field. Sections 1–5 essentially compute and estimate the density of the densest (resp., thinnest) packing (resp., covering) of the euclidean plane by congruent copies of a given convex disc. Lattice (resp., double-lattice) planar arrangements are studied as well from the point of view of the density. Sections 6–7 present the Rogers simplex upper bound for the densities of unit sphere packings in the d-dimensional euclidean space and present efficient random arrangements via the Minkowski-Hlawka theorem and some recent coding techniques. Section 8 describes Koebe's representation theorem (that connects circle packings to planar graphs) and gives a purely geometric proof of the Lipton-Tarjan separator theorem for planar graphs. The great list of remarks and exercises provides an additional list of related results suggesting further readings and research topics.

Part II of the book is devoted to the combinatorial geometry of points and lines, a fast-developing field that has been essentially created by Paul Erdős through his numerous deep questions. The eight sections on (9) Extremal Graph Theory, (10) Repeated Distances in Space, (11) Arrangements of Lines, (12) Applications of the Bounds on Incidences, (13) More on Repeated Distances, (14) Geometric Graphs,

1991 *Mathematics Subject Classification.* Primary 05-XX, 52-XX.

(15) Epsilon Nets and Transversals of Hypergraphs, and (16) Geometric Discrepancy present several of the major recent results of the field. Section 9 is a short elementary course on extremal graph theory, proving among others Turán's theorem and the Erdős-Stone theorem. Sections 10–12 dealing with the distribution of distances among n points in the plane (resp., in the d-dimensional Euclidean space) based on some techniques of extremal graph theory also estimate the maximum number of incidences between points and lines (resp., points and circles). Analogous questions are studied in section 13 under some special conditions (for example, the points are in convex position). This section presents also the recent counterexample of Kahn and Kalai to the famous conjecture of Borsuk on the partition of a set into "smaller" ones. Section 14 combines geometric and combinatorial ideas in order to study geometric graphs in an elegant way. The investigation of the general problem on geometric graphs has been initiated by Kupitz, Erdős and Perles and indicates the emergence of a new trend in combinatorial geometry. Section 15 introduces the Vapnik-Chervonenkis dimension in a very nice way, describing the machinery assigned to it that can be applied to several geometric problems leading to theorems like the nice theorem of Welzl on the stabbing number of a spanning tree. Section 16 studies a few sample problems of geometric discrepancy and proves the celebrated results of Beck, Matousek and Alexander combining some combinatorial and probabilistic methods with harmonic analysis and integral geometry. Part II of the book has a great list of exercises as well that is an excellent source of additional information and suggests several research problems for interested readers.

In short, the book is great reading, and I recommend it to students as well as to professional mathematicians.

Károly Bezdek
Eötvös University
E-mail address: kbezdek@ludens.elte.hu