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Leçons sur l'intégration et la recherche des fonctions primitives, by H. Lebesgue, second edition, Gauthier-Villars, Paris, 1928, xv + 342 pp.

A second edition of Lebesgue's book on integration after almost twenty-five years! Is it going to be a treatise on Lebesgue integration, following the lead of some recent books on this subject, or will it be in fact a second edition, i.e. a continuation and bringing up to date of the theme which dominated the first edition? The preface already gives enlightenment on this question. We find that Lebesgue has not yielded to the temptation of going into the details of the Lebesgue type of integration and its application, but is more interested in giving a logical continuation of the first edition of the book. He apologizes for omitting so many things which might be of interest, simply because they do not belong. The first edition is almost entirely contained in this second one, even to the extent of misprints, but the book has grown to almost three times its original proportions.

The new edition still forms one logical whole. The underlying problem and theme of the book is still to be found on the first page: "To find the functions $F(x)$ which admit as a derivative a given function $f(x)$." It is this theme with its variations, extensions and elaborations which forms the subject matter of the book. The author, in true composer fashion, begins with a simple statement of the theme, produces the harmony contained in the researches of Cauchy, which gives the wonderfully esthetic result that every continuous function is the derived function of another continuous function, carries it through the extension to Riemannian and related integrations, the introduction of Lebesgue integration, and as an added and extended feature, treating not only the derived functions but the upper and lower derivates. More and more complex become the variations as we approach the contribution made by Denjoy in his totalization, from which Lebesgue extracts in a triumphant fashion the important theorem on the possible distributions of the values of the derivates of a continuous function. Once again the theme starts simply and casually with a new kind of harmonic structure, the Stieltjes Integral puts in its appearance and through a series of complexities, one is led to a grand finale of the notion of derivatives with respect to functions of bounded variation and a totalization process as applied to these. As one applies a critical analysis to this composition as a whole, one cannot help but admire the skill and artfulness with which the entire structure is built, how each item contributes its share to the whole, how each consideration leads on to further results, and plays its own part in the sequel. It is indeed wonderful how the material drawn from the first edition seems in some farsighted way to have been a fitting preliminary to the later developments and takes its place as an integral and inseparable part of the whole.

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T. H. Hildebrandt

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