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Mathematics unbound: The evolution of an international mathematical research community, 1800–1945, by Karen Hunger Parshall and Adrian C. Rice (eds.), History of Math., vol. 23, American Mathematical Society/London Mathematical Society, Providence, RI, 2002, xxii + 406 pp., $85.00, ISBN 0-8218-2124-5

> "Prof. Klein tried hard to convince me to spend the next semester studying in Paris. He described Paris as a beehive of activity, especially among the young mathematicians, and thought that in view of this a period of study there would be most stimulating and profitable for me. He even joked that I should try to befriend Henri Poincaré and join him in drinking a toast to mathematical brotherhood." (David Hilbert to Adolf Hurwitz, 2 January 1886, Mathematiker-Archiv, Niedersächsische Staats- und Universitätsbibliothek Göttingen).

The universal brotherhood of mathematicians is an old-fashioned idea (hence its lack of gender neutrality). Hilbert appealed to it often, as at the 1928 Bologna ICM, where he (allegedly) said: "Mathematics knows no races. . . . For mathematics, the whole cultural world is a single country" (p. 366, quoted by Sanford Segal from Constance Reid's *Hilbert*). Whether or not Hilbert ever drank a toast to "mathematical brotherhood" with Poincaré, he did invite France's leading mathematician to Göttingen in 1909, greeting him with words that made his commitment to internationalism emphatically clear [16]. This volume's title thus conjures up an idealized image of communal mathematical activity that has been widely shared for a long time. It further suggests that by the end of the Second World War mathematicians were already well on their way to realizing a truly international research community.

Conference proceedings volumes seldom offer more than a loosely organized collection of individual contributions touching on a general theme. This compendium, which stems from a 1999 conference held at the University of Virginia, sets a far more ambitious goal. It strives to give a coherent picture of mathematical developments over a period of 150 years by addressing various manifestations of internationalization. This guiding theme is developed by the editors, Karen Parshall and Adrian Rice, in the opening chapter of *Mathematics Unbound*, which provides an "overview and an agenda" for the entire book. The agenda (about which more below) is inspired by a Baconian model for conducting historical research by means of a series of case studies. Thus, the subsequent chapters are designed to show (often by making use of abundant empirical data) how older national mathematical communities interacted just as newer ones were beginning to emerge, often under the influence of those that had already entered the international arena. Internationalization, the editors suggest, has taken on a wide variety of forms during the period 1800-1945. They view it as a process involving not only "the body of knowledge known as mathematics but also. . . the community of mathematicians and its practices" (p. 4).

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Several of the individual contributions to this volume are well worth reading, and some of them draw on previously published studies of major importance by the same authors (see [5], [7], [11], [12], [20], and [21]). Still, I doubt that many readers will want to plunge through this book from cover to cover. Its individual chapters are of mixed quality and hang together only rather loosely, as one would expect with any conference proceedings volume, however carefully edited. The case studies presented largely cluster around three geographical regions. Several deal with French mathematics and various strategies adopted by leading Parisians in the face of new research trends coming from Germany and Italy. Ivor Grattan-Guinness touches on several topics (real and complex analysis, mathematical physics, celestial mechanics) from the period 1820-1870, when French textbooks continued to exert a strong influence even as German and Italian mathematics were on the rise. Hélène Gispert discusses trends in professionalization and education during the ensuing period from 1870-1914, when nationalist sentiments were keenly felt. Her study shows how the Société mathématique de France, founded in 1872 in the wake of the Franco-Prussian War, was part of a larger movement to promote intellectual and moral resources in France. The themes addressed by Grattan-Guinness and Gispert are further illuminated in essays by Jesper Lützen and Thomas Archibald, who focus on the careers of Joseph Liouville and Charles Hermite, respectively. These two leading figures adopted strong international orientations that helped sustain the high level of French mathematics during the second half of the nineteenth century. The volume also contains essays by June Barrow-Green on Gösta Mittag-Leffler and by Aldo Brigaglia on Giovan Battista Guccia that stress their accomplishments as editors, respectively, of *Acta Mathematica* and the *Rendiconti del Circolo matematico di Palermo*, two important journals that adopted an international orientation from their inception.

Passing to the Asian theater of action, the book presents three substantial studies describing how Western mathematics influenced developments in Japan and China. Chikara Sasaki notes how traditional Japanese mathematics (*wasan*) quickly lost ground after the Meiji Restoration in 1868. After the founding of the University of Tokyo and the Tokyo Mathematical Society in 1877, several Japanese studied in Germany, where they became enamored with Prussian educational ideas. Takagi Teiji's mentor, Fujisawa Rikitaro, studied at the newly Germanicized Reichsuniversität Strassburg, where, according to Sasaki, he "acquired both the mathematical acumen and the political characteristics of his teacher," Elwin Bruno Christoffel. Fujisawa's mission ("Mathematics for the Nation!") apparently resonated well with the atmosphere at Strassburg University, which played a major part in realizing Bismarck's policies in the annexed territories of Alsace and Lorraine.

China's rise to prominence in the world of mathematics took a good deal longer. Joseph Dauben highlights the numerous waves of Western influence on mathematics in China during the nineteenth and early twentieth centuries. British and French culture gained a foothold in 1862 with the founding of Beijing's first "modern school", the Tongwen Guan, where Li Shanlan and Xi Gan taught mathematics. Dauben goes on to track other foreign factors, culminating with the impact of French and German institutions on higher mathematics in China during the 1920s and 1930s. As documented by Yibao Xu, this phase was accompanied and then followed by even more intense interactions between Chinese and American mathematicians, especially after World War II. Xu tells how S. S. Chern's first stay at

Princeton's Institute for Advanced Study opened the way for eight other young Chinese mathematicians who found their way to the IAS during the late 1940s.

Finally, returning to Europe, *Mathematics Unbound* culminates with essays by Reinhard Siegmund-Schultze and Sanford Segal on mathematics in Germany that focus on the Nazi period and its impact on internationalization, followed by a brief history of the International Mathematical Union by Olli Lehto which might be read as a follow-up to the essays on mathematics in France. Historians have seldom focused on the essential tension between national and international forces in the discourse of mathematicians and the policies of their constituent communities. Siegmund-Schultze, who has explored these issues in three probing studies [20], [21], [22] that cover the crucial period from 1918 to 1945, offers some interesting new reflections on internationalization in an essay entitled "The Effects of Nazi Rule on the International Participation of German Mathematicians". His account is based on two case studies: the first describes manifestly political factors that affected German participation at the Oslo ICM in 1936; the second recounts how Harald Geppert and Wilhelm Süss, two second-rate mathematicians but first-rate representatives of the Nazi Party, strove to maintain international ties during the Third Reich. Süss's highly successful career offers considerable food for thought. As *Führer* of the Deutsche Mathematiker-Vereinigung and the architect of the Mathematical Research Institute in Oberwolfach, Süss was ideally placed when it came time to reorient the German mathematical community after World War II. In an appendix, Siegmund-Schultze presents a chronology of events and biographical information bearing on the larger framework in which his two case studies are situated. Particularly revealing are the names compiled in Tables 2 and 3, the first listing German mathematicians who were leaders in international communication from 1933-1945, the second presenting a list of those who were hampered in this regard. A comparison of the two lists shows the strikingly weak correlation between international activism and a truly internationalist outlook in the spirit of Hilbert. Probably only Constantin Carathéodory could be counted as a full-fledged internationalist in this latter sense, whereas careerism and opportunism were important factors for mathematicians like Geppert, Süss, Wilhelm Blaschke, and Walter Lietzmann.

One way to approach this book would be to start with Jeremy Gray's essay "Languages for Mathematics and the Language of Mathematics in a World of Nations", which stands apart from the others in the volume both in its scope and originality. The world of nations was broadly represented at the 1900 Paris Exposition, depicted on the cover of *Mathematics Unbound* with the Eiffel Tower in the background. It was here during August that mathematicians convened for the Second ICM, mainly remembered today as the occasion for Hilbert's famous lecture on "Mathematical Problems" [8]. But Gray reminds us that this ICM was only one of some 200 congresses held in Paris during that year. These often chaotic events helped spur interest in developing an international language—Esperanto, Giuseppe Peano's Latino sine Flexione, and Ido were three possible contenders—to aid scientific discourse. In the meantime, several mathematicians had taken up the Leibnizian notion of a universal mathematical language, and Gray describes how this intense interest in formal logics (Peano, Frege, Schröder, Peirce, et al.) went hand in hand with the movement promoting a modern international language that could perform the same service as Latin during the Renaissance. Against this background, he then reconsiders the larger context of language, meaning, and mathematics as exemplified by Hilbert's work on the foundations of geometry and arithmetic. The issues Gray raises about efforts to formalize mathematical knowledge were clearly of central importance to the modernization of the discipline, a theme addressed by Herbert Mehrtens in his controversial study [13].

Several other authors discuss one or more of the ICMs. Olli Lehto's article on the formation of the IMU traces some of the infighting that led up to the first ICM held in Zurich in 1897. These events went hand in hand with an abortive effort to found an international organization of mathematicians. Like so many other endeavors, the five subsequent ICMs were dominated by national rivalries. Indeed, their venues—Paris (1900), Heidelberg (1904), Rome (1908), and Cambridge (1912)—precisely reflect the prestige each of the host nations was accorded within the European hierarchy. Mittag- Leffler, who had a huge personal stake in the ICMs from their inception, succeeded in lining up ICM VI for Stockholm in 1916. When World War I interceded, however, all talk about the international brotherhood of mathematicians quickly ceased, as did plans for the next ICM. It was against this acrimonious background that the IMU emerged in September 1920 under the auspices of the International Research Council. The IRC's first president, Émile Picard, was a staunch advocate of the view that subsequent international scientific affairs should be exclusively conducted by members of the "civilized" nations. Thus, with the quiet support of the "silent majority" among the Entente Powers, French scientists succeeded in ostracizing the Germans, Austrians, and Bulgarians from participation at meetings sponsored by the IRC. Mittag-Leffler and G. H. Hardy were among the few protesting voices. As Lehto notes, this policy of exclusion was adopted across the board in all fields of learning. This was the immediate context that produced the IMU, the first "international" organization of mathematicians, the larger context being the Versailles Treaty, with its Draconian measures and "war guilt" clause. The latter interpretation is mine; Lehto merely writes, "The German view... was clear and definite. Learned societies in France and Britain, supported by their governments, had founded the IRC for the purpose of undermining the position of German science."

Leaving aside the essays by Gray and Lehto, most of the other articles in *Mathematics Unbound* are concerned with specific conditions that prevailed in national communities or regional centers situated on the mathematical periphery, a term employed by Elena Ausejo and Mariano Hormigón in their brief essay on mathematics in Spain from 1700 to 1933. Several, on the other hand, offer analyses of leading mathematical journals in France, Britain, Italy, and Sweden. Evidence of an international "publication community" is presented by Sloan Evans Despeaux by counting foreign contributions to British journals. Brigaglia goes even further, arguing that those affiliated with the *Circolo matematico di Palermo* and who published in its *Rendiconti* constituted the "first international mathematical community"! These claims suggest that one can legitimately apply the term "community" to a group of authors who happen to publish their work in the same journal. This usage seems all the more dubious in light of the fact that nineteenth-century journals usually spanned the full gamut of pure and applied mathematics. Happily, Barrow-Green makes no such assertions for those who published in *Acta Mathematica*, the first truly international journal for mathematics.

In recent decades, national barriers and identities have ceased to act as constraints on mathematical research to the same degree as they did during the cold war era. Given the painful experiences with nationalism, fascism, and communism during the twentieth century, the twenty-first would finally seem ripe for advancing internationalism as a guiding ideology for all human affairs. If so, why should not mathematics be in the vanguard of this movement, just as it was during the French Revolution? The ideals of *liberté, égalité, fraternité* were often invoked in various contexts and with varied intentions by leading mathematicians like Georg Cantor, who proclaimed "the essence of mathematics lies in its freedom." Moreover, trends toward globalism and multiculturalism raise the issue of finding common cultural denominators. So what could be better suited to this task than the universal language of mathematics? Seen in this light, *Mathematics Unbound*, a book with plenty of promotional pizzazz, would seem to have found a promising topic for retrospective analysis. Still, its editors might have been more forthcoming about what this book is *not*.

From its back cover, we are told that this volume addresses the evolution of an international community from 1800-1945, one whose "development... was far from smooth and involved obstacles such as war, political upheaval, and national rivalries." In fact, only a few of the articles make any reference to such a community, nor is much evidence presented of efforts made by leading mathematicians to overcome the above-named obstacles. The editors further claim that "[d]uring this time, the practice of mathematics changed from being centered on a collection of disparate national communities to being characterized by an international group of scholars for whom the goal of mathematical research and cooperation transcended national boundaries." This assertion seems to me highly misleading given that national allegiances were especially strong and remained so at least through World War II. Nowhere between the covers of this book do we find any substantial support for this statement, though several authors provide plenty of evidence pointing the other way. As for "mathematical practice", this mainly took place within local research centers which were (and still are) far more important than "national communities". Clearly the universe of "mathematical nations" grew, but so did the importance of national rivalries, a pattern familiar from historical studies of the physical sciences that chronicle the activities of figures like Einstein, H. A. Lorentz, and Madam Curie. Parshall and Rice insist, however, that ideology should be seen as largely irrelevant for mathematics, a claim that flies in the face of Herbert Mehrtens's findings in [13] and [14], works mentioned nowhere in this volume. In these studies, Mehrtens describes the ideological cross-currents of mathematical discourse in Germany between 1900 and 1925 as a prelude to the emergence of Ludwig Bieberbach's version of "Deutsche Mathematik" in 1933. Central figures in this story include Hilbert, Klein, Hausdorff, Brouwer, and Weyl, none of whom receive more than scant attention in *Mathematics Unbound*.

Beyond these weaknesses, this volume simply cannot deliver on the theme advertised in its subtitle, because the case studies it presents leave so many important areas untouched. How can one describe "the evolution of an international mathematical research community", itself a dubious construct, and say virtually nothing about mathematics in Germany during the critically important period from 1900 to 1933 or about developments in the United States from 1900 to 1945? During the latter period important mathematical research took place at several leading centers throughout Europe: Paris, Berlin, Göttingen, Rome, and Cambridge. By 1920 Harvard and Princeton had emerged as the two leading mathematics faculties in the

United States, and by the 1930s the U.S. had begun to attract talented mathematicians from around the world. Prior to World War I many aspiring mathematicians studied in Germany, and several foreigners spent a good deal of time at leading German research centers. During the heyday of Klein and Hilbert, Göttingen was filled with visitors from the United States and Eastern Europe (two prominent Japanese, Takagi and Yoshiye Takuzi, are discussed in the article by Chikara Sasaki). During the 1920s, the Rockefeller Foundation financed the building of Göttingen's Institute of Mathematics as well as the Institut Henri Poincaré in Paris (see [22]) as part of a major effort to support leading international scientific centers. Soon afterward, Abraham Flexner succeeded in launching a comparable center in the U.S. with Princeton's Institute for Advanced Study. This shift clearly had much to do with larger political events, including the precarious situation of German scholarship during the Weimar Republic and the anti-Semitic policies of the Nazi regime (briefly discussed by Segal). Under the leadership of Oswald Veblen and his colleagues, Princeton emerged as the world's premier internationalized mathematical research community, as dramatized by the wealth of talent that congregated when its mathematicians hosted the 1946 conference on "Problems of Mathematics" (see [4], pp. 309-360]). None of these mainstream developments receives adequate attention in *Mathematics Unbound*, which contains nothing at all about mathematics in Hungary, Poland, and Russia (including the Soviet Union). Even information about the early International Congresses of Mathematicians is scarce, most of it tucked away in the book's final chapter, a truly puzzling editorial lapse given that the ICMs were the only international venue in which mathematicians regularly convened.

As for mathematical knowledge itself, only a few of the articles (for example, those by Grattan-Guinness, Lützen, and Gray, as well as Laura Martini's paper on Galois theory in Bologna) give any information about specific research results. Even more glaring is the lack of pertinent information about mathematics produced after 1900. None of the authors make use of the massive documentation compiled by a truly international team of experts for the volumes of the *Encyklopädie der mathematischen Wissenschaften* between 1894 and 1935, and yet no other single source provides such a wealth of information relevant to the specific national research traditions of this period: French contributions to analysis, Italian work on algebraic geometry, number theory in the German tradition, and the eclectic style of British research on mathematical physics. *Mathematics Unbound* has little to say about these and other mainstream developments in fields like topology, functional analysis, and modern algebra. A few of its authors, however, try to "privilege" the work of prolific American mathematicians like L. E. Dickson or G. A. Miller, figures who were completely overshadowed by leading contemporaries Birkhoff, Veblen, Lefschetz, and Zariski, none of whom receive more than passing mention in the book. Almost no use is made of the rich source material on local research centers in the United States compiled in [2], [3], and [4] or several relevant studies in [6]. Nor do any of the articles in *Mathematics Unbound* touch on the many important themes raised by Thomas Hawkins in his monumental history of the theory of Lie groups from 1869 to 1926 [9] (see my review [19]). In short, this book skirts around nearly all the mainstream developments that took place after 1870.

None of this bodes well for the agenda the editors have in mind, which calls for more historical case studies of "internationalizing impulses" in other national mathematical communities around the globe. Their main rationale for promoting such studies apparently stems from dissatisfaction with the way in which internationalization has been handled by historians of science, many of whom have generally ignored mathematics and its practitioners. As regrettable as this may be, I fail to see how more case studies of national communities along the lines of those in this volume will shed any significant light on the mathematical research practices of the twentieth century. What is really needed are historical studies, like Hawkins's work on Lie groups, that show how mathematics actually gets made. Hawkins's study provides a splendid case study along these lines, one that offers many insights into mathematical research practices and the local communities that fostered them. It also illustrates the highly contingent nature of mathematical knowledge by demonstrating the crucial importance of the research traditions embedded in local cultures. Thus, in describing the work of Killing, Frobenius, and Schur, Hawkins emphasizes the ethos shared by Berlin's algebraists. He points out some of the special characteristics of Lie's Leipzig school and contrasts its goals and vision with the views held by leading Parisians, including Élie Cartan. Turning to Weyl's pioneering work on global Lie theory, he links this with major currents within Hilbert's research legacy, which he traces back to a conceptual approach championed by Riemann. What emerges from all this is a coherent picture of how an impressive mathematical theory emerged over a period of more than fifty years. Moreover, Hawkins shows how the work of the principal architects of Lie theory—Lie, Killing, Cartan, and Weyl—reflects deep-seated commitments to the ideals that were operant in their respective research communities. If historians of mathematics are looking for an agenda for future research, I cannot imagine a better model than Hawkins's book.

The editors of *Mathematics Unbound* write as if professionalization and the institutional and organizational trappings of "national communities" played a major part in the making of modern mathematics. A closer analysis reveals, however, that while national organizations definitely promoted mathematical research, they had relatively little to do with actually producing it. Throughout the period 1800-1945 most of the significant research activity took place within local centers, not at national meetings. True, mathematicians have often found ways to communicate and even to collaborate without being in close physical proximity. Nevertheless, *intense* cooperative efforts have normally necessitated an environment where direct, unmediated communication was possible. This type of arrangement, the modern mathematical research school, persisted in various forms throughout the nineteenth and twentieth centuries (see [18]). As with other fields, mathematical schools accompanied a general trend toward specialization in scientific research. Recently, historians of science have tried to understand how the kind of locally gained knowledge produced by research schools becomes "universal", a process that involves analyzing all the various mechanisms that produce consensus and support within broader scientific networks and communities. Similar studies of mathematical schools, however, have been lacking, a circumstance no doubt partly due to the prevalent belief that mathematical knowledge is from its very inception universal and hence stands in no urgent need to win converts. This simplistic picture of mathematics as "pure ideas" was skillfully satirized by Davis and Hersh in [1]. As I have argued in [17], such a viewpoint seriously hampers any serious attempt to understand mathematical practices historically. As a relatively stable element within the complex, fluctuating picture of mathematical activity over the last two centuries, research schools and centers offer historians a convenient category for better understanding *how* and *why* mathematicians produce their work rather than compiling statistics based on publications in the journal literature.

References

[1] Phillip Davis and Reuben Hersh. *The Mathematical Experience*. Birkhäuser, Boston, 1981. MR **82i:**00020

[2] Peter Duren et al., eds. *A Century of Mathematics in America, Part 1*, History of Mathematics, vol. 1, American Mathematical Society, Providence, R. I., 1988. MR **90a:**01064

[3] Peter Duren et al., eds. *A Century of Mathematics in America, Part 2*, History of Mathematics, vol. 2, American Mathematical Society, Providence, R. I., 1989. MR **90a:**01065

[4] Peter Duren et al., eds. *A Century of Mathematics in America, Part 3*, History of Mathematics, vol. 3, American Mathematical Society, Providence, R. I., 1989. MR **90f:**01044

[5] Hélène Gispert. *La France mathématique: La Société mathématique de France (1870-1914).* Cahiers d'histoire et de philosophie des sciences **34** (1991): 11-180. MR **96f:**01040

[6] Catherine Goldstein, Jeremy Gray, Jim Ritter, eds. *L'Europe mathématique, Mathematical Europe*. Édition de la Maison des sciences de l'homme, Paris, 1996. MR **2002c:**01003

[7] Ivor Grattan-Guinness. *Convolutions in French Mathematics, 1800-1840.* 3 vols., Birkhäuser, Basel, 1990. MR **92h:**01032a, MR **92h:**01032b, MR **92h:**01032c

[8] Jeremy Gray. *The Hilbert Challenge*. Oxford University Press, Oxford, 2000. MR **2002c:**01037

[9] Thomas Hawkins. *Emergence of the Theory of Lie Groups*. An Essay in the History of Mathematics, 1869-1926. Springer-Verlag, New York, 2000. MR **2001f:**01031

[10] Solomon Lefschetz et al., eds. *Problems of Mathematics*. Princeton University Bicentennial Conferences, Series 2, Conference 2, 1947, reprinted in [3], pp. 309-334. MR **8:**556k

[11] Olli Lehto. *Mathematics Without Borders: A History of the International Mathematical Union*. Springer-Verlag, New York, 1998. MR **98m:**01039

[12] Jesper Lützen. *Joseph Liouville: Master of Pure and Applied Mathematics*. Springer-Verlag, New York, 1990. MR **91h:**01067

[13] Herbert Mehrtens. *Moderne Sprache, Mathematik. Eine Geschichte des Streits um die Grundlagen der Disziplin und des Subjekts formaler Systeme*. Suhrkamp Verlag, Frankfurt, 1990. MR **92g:**01005

[14] Herbert Mehrtens. *Modernism vs. Counter-Modernism, Nationalism vs. Internationalism: Style and Politics in Mathematics 1900-1950*. [6], pp. 518-529. MR **2002c:**01003

[15] Mary Jo Nye, ed., The Cambridge History of Science. Volume 5: *The Modern Physical and Mathematical Sciences*. Cambridge University Press, Cambridge, 2003.

[16] David E. Rowe. *David Hilbert on Poincaré, Klein, and the World of Mathematics*. Mathematical Intelligencer **8**, no. 1 (1986): 75-77. MR **87j:**01038

[17] David E. Rowe. *New Trends and Old Images in the History of Mathematics*. Vita Mathematica. Historical Research and Integration with Teaching, ed. Ronald Calinger, MAA Notes Series, vol. 40. Mathematical Association of America, Washington, D.C., 1996, pp. 3-16. MR **97d:**01002

[18] David E. Rowe. *Mathematical Schools, Communities, and Networks*. [15], pp. 113-132.

[19] David E. Rowe. Review of [9]. Notices of the AMS **50** no. 6 (2003), 668-677.

[20] Reinhard Siegmund-Schultze. *Mathematische Berichterstattung in Hitlerdeutschland: Der Niedergang des Jahrbuchs über die Fortschritte der Mathematik (1869-1945).* Vandenhoeck & Ruprecht, Göttingen, 1993. MR **95f:**01041

[21] Reinhard Siegmund-Schultze. *Mathematiker auf der Flucht vor Hitler*. Dokumente zur Geschichte der Mathematik, Bd. 10, Vieweg, Braunschweig, 1998. MR **2000f:**01022

[22] Reinhard Siegmund-Schultze. *Rockefeller and the Internationalization of Mathematics Between the World Wars*. Birkhäuser, Basel, 2001. MR **2002g:**01019

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