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Computation and complexity in economic behavior and organization, by Kenneth R. Mount and Stanley Reiter, Cambridge Univ. Press, Cambridge and New York, 2002, x+237 pp., $55.00, ISBN 0-521-80056-0

More and more, my fellow economists are coming to recognize that all forms of economic activity entail *computation.* Given the way mathematics has emerged as the preferred language of economic theory, it is perhaps surprising that this aspect of economics has only recently begun to be treated formally. The late emergence of the computational theme in economics may be a consequence of the fact that the notion of computational limits is itself relatively new in mathematics. Gödel and Turing first discovered limits to what can be proved or computed in the 1930s, and formal theories of computational complexity and algorithmic information only began to emerge in the 1960s [1], [4]. Model-building styles in economics have been slow to catch up.

Since the beginnings of complexity theory in mathematics, it has been recognized that computational limits have economic implications. When Herbert Simon introduced "bounded rationality" in the 1950s, he had in mind inherent limits to human information-processing capabilities: "Broadly stated, the task is to replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist" ([10], p. 99). In addition to the kinds of intrinsic human psychological limitations Simon emphasized, computational complexity issues are obviously important for many of the practical problems faced by business firms in their day-to-day activities. Well-known examples include the Traveling Salesman Problem (TSP), the Production Planning Problem, and many others included in Garey and Johnson's long list of NP-complete problems [4]. More recently, economists have come to realize that any theory of decision-making that is supposed to weigh costs and benefits is incomplete without taking account of the costs of the decision-making process itself.

What Mount and Reiter (hereafter MR) have done is to develop a measure of the "complexity" of economic computations relative to a set of elementary or "primitive" functions and set of directed graphs. Any computation that can be expressed as a superposition of the primitive functions can be represented by a corresponding graph structure, and MR show that only a particular type of graph (ordered trees) is required. The measure of complexity is then simply the height of the ordered tree (the length of the longest path in the tree). The model involves some interesting mathematics - results about superpositions of functions are the content of Hilbert's 13th problem, nomography, and Leontief's Theorem. The MR setup provides a natural way to think about computations that are performed by humans and machines in conjunction, by specifying that some of the primitive functions are those that are easy for humans (like handwriting recognition) but difficult for computers.

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By including among the primitive functions "what humans do", MR deliberately avoid thorny issues such as whether physical reality is discrete or continuous, whether brains are simply Turing machines, and so forth. MR's unwillingness to fish in these troubled philosophical waters has the practical advantage of allowing them to proceed directly to examples of the usefulness of their model for economic theorizing. It also means that they are able to ground the notion of "complexity" in a cultural context - computations that are easy for a person today who has access to a fast PC running Mathematica would have been difficult if not impossible for Euclid or Archimedes. Nevertheless, the MR definition of complexity begs the question of whether the processes being carried out by brains or business organizations are somehow fundamentally constrained by the limits of algorithmic computation.

This question matters because the role of theory in an empirical science like economics is to place restrictions on what can happen. In classical physics, for example, the conservation laws (plus Newton's inverse square law of gravitation) provide the foundation for testable hypotheses about the orbits of planets and other mechanical systems. Economics has traditionally sought its defining restrictions through the imposition of a strict kind of rationality on the agents who populate its models. (Optimization of production and maximization of utility can be thought of as particular consequences of rationality.) Unfortunately, modern economic theory shows that rationality alone does not provide enough structure to model real-world phenomena adequately. The classical tradition in economics stretching back to Adam Smith and culminating in neoclassical General Equilibrium Theory aims to derive the essential characteristics of the market economy from underlying fundamentals of tastes and technology, but even though rationality is enough to guarantee the existence of an equilibrium, it cannot rule out multiple equilibria and unstable dynamics [5], [8]. A similar failure is the inability of rationality alone to characterize the outcome(s) of multiplayer games without additional assumptions about context, timing, and even the nature of the solution concept [3], [9].[1]

Given that rationality alone is not sufficient to construct a suitable economic theory, it is possible that the limits that emerge from the theory algorithmic computation might provide at least some of the requisite restrictions, provided economic activity can properly be interpreted as algorithmic computation. This is why the deeper questions of "what is an algorithm?" and "can humans (and their organizations) be represented as Turing machines?" carry weight. Mathematical impossibility or intractability results have been shown to carry over to a range of economic models that are quite far removed from simple applications like the TSP. Even if the restrictions that arise from computational limits take the form of negative assertions, these might still have practical significance for economists by steering them away from otherwise appealing concepts such as "rational expectations" or fully optimized production that cannot be achieved in practice [2], [11].

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[1] As Shubik puts it, "[b]eginning with their [von Neumann and Morgenstern's] work, a surprisingly large number of ingenious and insightful solution concepts for n-person games have been proposed by many different authors. Each solution probes some particular aspect of societal rationality, that is, the possible, proposed, or predicted behavior of rational individuals in mutual interaction. But all of them have had to make serious compromises. Inevitably, it seems, sharp predictions or prescriptions can be had only at the expense of severely specialized assumptions about the customs or institutions of the society being modeled. The many intuitively desirable properties that a solution ought to have, taken together, prove to be logically incompatible" ([9], p. 2).

Hence MR have provided *a* story about computation in economics, but not *the* computational foundation of economic theory. This does not mean that what MR have done is not important. The bounds they derive for the complexity of functional calculations have meaningful economic implications, as shown by several of the examples they work out in detail. It is certainly useful to know whether a participant in a game has an advantage by having greater computational ability, or to examine how information-processing limitations constrain the solutions to coordination problems in firms and markets. Nevertheless, many of the MR results are specific to particular specifications of goals, technologies, or game structures. As insightful as the examples may be, they are some distance from being a fully general economic theory grounded in computation. Nevertheless, economists (and mathematicians) should welcome the kind of research exemplified by MR's book. MR and others have shown conclusively that economic outcomes are contingent on the nature of the computational processes that are embedded in economic activity (see also [6], [7], and [12]). In the future, computational issues seem likely to move towards center stage in economics. It may be, however, that a reconstructed theoretical foundation for economics that includes computational limits may not look very much like the grand conception of neoclassical economics - a concise representation of the welter of economic life as the expression of a few basic principles. Human reality is just too complex for that. Our understanding of social and economic activity will always be limited in (at least) the same ways mathematical knowledge is limited. All the disciplines of modern thought still reverberate to Hilbert's stirring challenge - "Wir müssen wissen. Wir werden wissen." Hilbert's vision may not be attainable, but economists, like mathematicians, can recognize the wisdom of acknowledging that there are some things we cannot know.

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