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Most honorable remembrance: The life and works of Thomas Bayes, by Andrew I. Dale, Springer-Verlag, New York, 2003, xxiv + 668 pp., $99.00, ISBN 0-387-00499-8

1. The Reverend Thomas Bayes

When the Reverend Thomas Bayes died in 1761, he left among his effects a manuscript entitled "An essay towards solving a Problem in the Doctrine of Chances". Bayes's friend and intellectual executor Richard Price edited the manuscript for publication; it appeared three years later in the *Philosophical Transactions of the Royal Society of London*. The problem that Bayes set himself, Price tells us, was

> to find out a method by which we might judge concerning the probability that an event has to happen, in given circumstances, upon supposition that we know nothing concerning it but that, under the same circumstances, it has happened a certain number of times, and failed a certain other number of times.

Seldom has so apparently modest a contribution had so great an impact, for with the publication of this paper Bayes became the eponymous founder of the field of "Bayesian statistics", thus winning for himself undying fame.

So-called inverse or Bayesian methods have had a chequered history. Championed by Laplace a decade later, inverse methods reigned supreme for more than half a century, and even after they began to be seriously challenged in the mid-nineteenth century, they remained predominant – at least in terms of the foundations of the subject – until a near-simultaneous assault on two fronts by Fisher and Neyman in the 1920s and 1930s caused the Bayesian approach to go into eclipse. But the post-war period saw a revival, due in part to the efforts of Frank Ramsey, Bruno de Finetti, Harold Jeffreys, I. J. Good, L. J. Savage, Howard Raiffa and Robert Schlaifer (among many others). Today the Bayesian viewpoint and Bayesian methods are once again a flourishing branch of modern statistical theory and practice.

In the Bayesian approach, probabilities evolve over time; *initial* (or *prior*) probabilities become *final* (or *posterior*) probabilities as new information is received, incorporated via conditioning. Our initial opinions, enshrined in our prior probabilities, can be of great importance even after we receive new information: if we are playing poker and our opponent deals himself a royal flush, then it makes a difference whether he is Doc Holliday or the Archbishop of Canterbury.

But how does one choose the prior probabilities the theory demands? This has always been a lightening rod for criticism of the Bayesian approach. Laplace and his disciples (the latter sometimes invoking the "principle of indifference") usually assigned so-called uniform priors to the parameters being estimated, a practice often sharply attacked on a number of grounds. Part of the importance of the contributions of Ramsey (in his posthumously published 1926 essay *Truth and Probability*) and de Finetti (most notably in his 1937 paper *La prévision: ses lois logiques, ses sources subjectives*) was that they provided a new theoretical superstructure that permitted the Bayesian approach to break out of this straightjacket.

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2. Bayes's essay

An enormous amount has been written about Bayes's paper itself. One of the most fascinating aspects of the essay is the degree to which Bayes was obviously aware of many of the difficulties inherent in the problem. (The British statistician R. A. Fisher thought this was the reason Bayes did not have his work published during his lifetime.) For example, unlike Laplace, Bayes did *not* assume a uniform distribution for the unknown probability p. Instead, he began by considering a physical randomization model: throwing a ball on a table. Bayes assumes that an initial ball is thrown and its position relative to one of the sides of the table noted. Then n more balls are thrown, and one keeps track of the number S_n falling to the right of the initial ball, relative to the side chosen. (Here and below Bayes's argument is translated into modern terminology and notation.)

On the assumption that the initial ball is equally likely to fall anywhere along the side, Bayes shows that the probability that k balls fall to the right is

$$P[S_n = k] = \binom{n}{k} \int_0^1 p^k (1-p)^{n-k} dp = \frac{1}{n+1}.$$

That is, the number of successes is equally likely to be any of the possible values of k. Bayes then turns to the case of "an event concerning the probability of which we absolutely know nothing antecedently to any trials made concerning it" and argues that *here too* $P[S_n = k] = (n+1)^{-1}$ (if you like, this is Bayes's *definition* of such an event), and on this basis he concludes that the prior probability of p is the uniform distribution dp. Bayes does not expand on this latter identification (he says only that it "seems to appear from ... the consideration" just noted), but it can be seen as a simple consequence of the Hausdorff moment theorem: if we replace dp by a general prior $d\mu(p)$ and set $k = n$, we conclude that the moments of $d\mu$ are

$$\mu_n = \int_0^1 p^n d\mu(p) = \frac{1}{n+1}.$$

Since the measure $d\mu$ is concentrated on a compact set, it is uniquely determined by its moments; and since dp has precisely these moments, it follows that $d\mu(p) = dp$. Ingenious indeed!

3. When and why?

Like his predecessor James Bernoulli, Bayes not only perished before he was published but appears to have withheld his result for quite some time. A notebook of Bayes that has been discovered contains a proof of one of the rules in his essay; although undated, the entry occurs between two other entries dated 1746 and 1749. Consistent with this timeframe is a comment by David Hartley in his *Observations on Man* of 1749, reporting that "[a]n ingenious friend has communicated to me a Solution of the inverse problem," that is, the problem Bayes had considered. (A small cottage industry of skeptics has sprung up around this passage in Hartley, arguing either that the reference is to someone other than Bayes or that the reference is to some result other than Bayes's. This has always seemed to this author a real stretch. Dale himself was once one of the skeptics, but in this book he appears to recant, apparently in part on the basis of new evidence he presents.)

But what led Bayes in the period 1746 to 1749 to address the fundamental problem of induction? Here an obvious answer suggests itself. The Scottish philosopher

David Hume had questioned the logical justification for inductive inference in both his *Treatise* of 1739 and his *Enquiries* of 1748. The former, to use Hume's phrase, "fell still-born from the press," but the *Enquiries* were widely read and debated, and it is possible that Hume's book was directly responsible for Bayes's interest in the subject. What is indisputable is that Bayes's friend Price saw the significance of the one for the other: much of a lengthy appendix Price added at the end of the essay, explaining its application to the problem of inductive inference, reads as a direct attack on Hume (see, e. g., Gillies, 1987 [3]; Zabell, 1996 [9]).

4. In search of Bayes

Despite the interest that his essay has elicited for the last two and a half centuries, the Reverend Thomas Bayes has himself, until relatively recently, remained an obscure figure. When Bayes's paper was reprinted in 1958 in the journal *Biometrika*, the English statistician George Barnard wrote an accompanying biographical note less than three pages long, observing:

> And yet such are the vagaries of historical records, that almost nothing is known about the personal history of the man.

This lack of information, however, was only apparent rather than real. If one is enterprising enough, there is a surprising amount of information about the past squirreled away in archives, personal libraries, public record offices, and so on. All that is needed to root it out is initiative, persistence, and sometimes a bit of luck. Robin Winks's classic collection *The Historian as Detective* chronicles a number of fascinating instances; an example in the history of mathematics is Constance Reid's *The Search for E. T. Bell* (1993) [6].

So it should not really be surprising that in the last several decades a number of enterprising historians of mathematics have become interested in Bayes, and, it turns out, there is in fact a considerable amount of information about the man that can be unearthed. First a notebook was discovered in the offices of the Equitable Life Assurance Society in London (Holland, 1962 [5]), then records at Edinburgh University dating back to the days when Bayes studied there, then drafts of papers in the private collection of the Earl of Stanhope, and so on.

For those interested in Bayes's biography, there are now two outstanding places to turn. First, there is a wonderful paper by David Bellhouse (2003) [2]. Bellhouse has long been interested in Bayes and uncovered the manuscripts in the Stanhope collection. His paper, forty pages long, sets out very carefully exactly what is now known about Bayes. The focus of the paper is primarily Bayes's life rather than an analysis of his work.

At the other extreme (in terms of length) there is Andrew Dale's book, the subject of this review.

5. Most honorable remembrance

Dale's book is 668 pages long (!); gone are the days when a George Barnard could summarize the basic facts about Bayes in less than three pages. There are chapters devoted to Bayes's ancestry, his life, "momento mori" (information about wills and burial grounds), and of course his work: two chapters on books that are attributed to him, three on papers in the *Philosophical Transactions*, one on his notebook, and two on unpublished letters and manuscripts. This is a very personal book, as will no doubt strike the reader from its title onwards, and it is peppered with asides and quotations from Horace, Pepys, Locke, Pope, Hazlitt, Huxley, Omar Khayyam, Orrick Johns, and many others. It also is a considerable work of scholarship: there are nearly a hundred and twenty pages of endnotes, the bibliography is nearly forty pages long, and the coverage is exhaustive.

There are several ways to read this book. Those interested primarily in Bayes's life and times should start with the first three chapters, then skip forward to the ninth and tenth chapters (discussing Bayes's correspondence), and conclude with the last chapter (describing Bayes's grave and his will).

The rest of the book can be read in a variety of different ways. Each chapter discusses one of Bayes's published or unpublished books or papers, starting with a brief introduction, followed by the complete text of the book or paper and ending with a detailed commentary. Many readers will probably be most interested in the chapter discussing Bayes's essay; although the text of the essay itself is nowadays readily available via JSTOR, Dale provides an extensive apparatus in the form of commentary and notes. (There are, of course, a number of other detailed commentaries on the essay. For two other excellent discussions of its contents, see Stigler, 1986, pp. 122 – 131 [7]; Hald, 1998, Chapter 8 [4].)

The remaining chapters of the book are of somewhat more specialized interest. Chapter 4 contains Bayes's tract on *Divine Benevolence*. Although the tract was published anonymously, there is no doubt whatever about the attribution: Richard Price himself (in his *A Review of the Principal Questions in Morals* of 1787) tells us that Bayes was the author. Although an interest in the theological debates of eighteenth century England must surely be considered an acquired taste, both Bayes and Price took these issues very seriously, and the tract, together with Dale's commentary, gives an interesting sense of the period from this vantage point.

Chapter 5 is devoted to an anonymous tract defending the "doctrine of fluxions" (that is, the infinitesmal calculus) against Bishop George Berkeley's attack in his book *The Analyst* of 1734. Dale's commentary is of interest, but few are likely to want to go through the tract itself! (It is usually attributed to Bayes on the authority of the nineteenth century English mathematician Augustus de Morgan, himself an accomplished antiquarian.)

Chapter 6 then goes on to discuss Bayes's paper concerning the nature of Stirling's asymptotic series for $\log n!$. Ironically, this brief (two page) paper also appeared posthumously in the same volume of the *Philosophical Transactions* as Bayes's more famous essay. Short as it is, this paper provides a glimpse of the mathematical power Bayes must have possessed, an ability known to be responsible for his election to the Royal Society in 1742. In it Bayes notes – for the first time – the divergent nature of the asymptotic series for $\log n!$. This is impressive because, as Bayes observes, "some eminent mathematicians" (including, it might be remarked, de Moivre himself) had mistakenly thought the series convergent.

The remaining chapters in Dale's book are likely to be of quite limited interest to most readers, but here too many may still enjoy browsing through them, if only for the occasional disgression. This brings us, however, to the primary weakness of the book: it suffers from a lack of discipline; too much is quoted, and not always to enough point. (For example, the paragraph on p. 43 about the effect of Charles Lamb's stutter on his career could easily have been omitted without loss, and if included should have appeared as a footnote instead.) One result is that it sometimes seems unnecessarily difficult to track facts down, such as the date of

Bayes's election to the Royal Society or the date of his death. The index entry for Bayes ("Bayes, Thomas, *presque partout*") did not prove helpful here.

This is, as noted, a very personal book. Dale has a distinctive sense of humor, relishes the arcane, and seems to enjoy the byways as much or more than the main road. Each reader will in turn react to the book's style in their own personal way. But what is indisputable is that Dale's book is an important contribution to the history of statistics.

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