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MATHEMATICS: ART AND SCIENCE

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We all know what we like in music, painting or poetry, but it is much harder to explain why we like it. The same is true in mathematics, which is, in part, an art form. We can identify a long list of desirable qualities: beauty, elegance, importance, originality, usefulness, depth, breadth, brevity, simplicity, clarity. However, a single work can hardly embody them all; in fact some are mutually incompatible. Just as different qualities are appropriate in sonatas, quartets or symphonies, so mathematical compositions of varying types require different treatment. Architecture also provides a useful analogy. A cathedral, palace or castle calls for a very different treatment from an office block or private home. A building appeals to us because it has the right mix of attractive qualities for its purpose, but, in the end, our aesthetic response is instinctive and subjective. The best critics frequently disagree.

Since mathematics is partly art and partly science (corresponding roughly to the traditional pure/applied distinction), we may feel that appreciation of "good mathematics" should be less subjective and based on firmer logical principles. That is probably true to some extent, but as long as mathematics remains a human endeavour and is not reduced to a pure computer calculation, we cannot ignore the human factor and the welcome variety that this entails. So what follows are my personal preferences and make no claim to universality. In fact I glory in the diversity of mathematics and the lack of a uniform straightjacket. This is what makes it live.

The first point I would like to make is the distinction between the strategic or global aspect and the tactical or local one. The first involves the development of a theory, an architectural structure, and can be admired because of its importance, the breadth of its applications, as well as its rational coherence and naturality. The spectral theory of self-adjoint operators, Grothendieck's K-theory or the representation theory of Lie groups would be examples.

The tactical aspect is concerned with the minutiae of the argument: short, succinct proofs of individual theorems, such as the irrationality of $\sqrt{2}$ or, on a grander scale, one of the simpler versions of the Bott periodicity theorem. Here the appeal

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lies in the simplicity and beauty of both the result and the argument. If theory is the role of the architect, then such beautiful proofs are the role of the craftsman. Of course, as with the great renaissance artists, such roles are not mutually exclusive. A great cathedral has both structural impressiveness and delicate detail. A great mathematical theory should similarly be beautiful on both large and small scales.

The second criterion that appeals to me is originality. I like to be surprised. The argument that follows a standard path, with few new features, is dull and unexciting. I like the unexpected, a new point of view, a link with other areas, a twist in the tail. Again, originality can take global or local form. A whole theory can be breaking new ground, as with the cobordism theory of René Thom, where the audacity of the conception was matched by its spectacular applications. But originality can also occur at the detailed technical level, as with the evaluation (by Euler?) of the standard Gaussian integral. I personally have always had a soft spot for the miraculous computation of contour integrals by residues, a surprise when you first meet it, though now embodied in general theory.

The grandest surprises, and the ones that impress me most, are the unexpected links between apparently quite different parts of mathematics. Classical ones include the use of complex analysis in number theory or the way the conic sections of the Greeks appeared in the Kepler-Newton theory of planetary orbits. More recent examples would be the link between water waves and spectral theory that lies behind the KdV equation or the work of Vaughan Jones and Witten connecting polynomial invariants of knots with statistical physics and quantum field theory.

This last example is in fact the tip of an iceberg, just one instance of a vast number of beautiful and unexpected links between geometry and physics (counting algebraic curves, Donaldson invariants of 4-manifolds, homology of moduli spaces). I find this whole area quite overwhelming in its grandeur: it opens up so many new avenues, it has so many mysteries, and it unites large parts of mathematics and physics in a totally new way. It re-invigorates mathematics and is an encouragement for the future.

Perhaps I should end with a word about applied mathematics, in the broad sense. Much of mathematics was either initiated in response to external problems or has subsequently found unexpected applications in the real world. This whole linkage between mathematics and science has an appeal of its own, where the criteria must include both the attractiveness of the mathematical theory and the importance of the applications. As the current story of the interaction between geometry and physics shows, the feedback from science to mathematics can be extremely profitable, and this is something I find doubly satisfying. Not only can we mathematicians be useful, but we can create works of art at the same time, partly inspired by the outside world.

About the author

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