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Noncommutative geometry and particle physics, by Walter D. van Suijlekom, Mathematical Physics Studies, Springer, Dordrecht, 2015, xvi+237 pp., ISBN 978-94-017-9161-8 (hardcover), 978-94-017-9162-5 (electronic), US $69.99 (hardcover), US $54.99 (electronic)

The use of noncommutative geometry (NCG) as a tool for constructing particle physics models originated in the 1990s [9, 11]. The main idea can be heuristically regarded as similar to the idea of "extra dimensions" in String Theory, except for the fact that the nature and scope of these extra dimensions is quite different. In the NCG model one considers an "almost commutative geometry", which is a product (or locally a product in a more refined and more recent version [4]) of a four-dimensional spacetime manifold and a space of inner degrees of freedom, which is a "finite" noncommutative space, whose ring of functions is a sum of matrix algebras. According to the choice of this finite geometry, one obtains different possible particle contents for the resulting physics model. The physical content is expressed through an action functional, the *spectral action* [5], which is defined for more general noncommutative spaces, in terms of the spectrum of a Dirac operator.

In the case of an almost-commutative geometry, the asymptotic expansion of the spectral action, for large energy scales, recovers the usual physical action functionals. In particular, one finds a (modified) gravity action functional that includes the Einstein–Hilbert action of General Relativity, coupled (nonminimally) to the matter sector, described through a Lagrangian. For particular choices of the finite geometry, the latter recovers the full Lagrangian of the Standard Model of elementary particle physics, extended with right-handed neutrinos with Majorana mass terms, [7] and [2, 10].

Further modifications of the noncommutative geometry approach to particle physics were recently developed, which include scalar fields, used either to produce a correct value of the Higgs mass [6, 13] or as models for inflationary cosmology [19]. Some forms of grand unification can also be accommodated by this type of model [8, 14] and an approach toward the Minimally Supersymmetric Standard Model, within the NCG framework, was developed in [3]. There are also interesting results regarding renormalization in spectral action models; see for instance [21]. Focusing on the gravity part of the model, on the other hand, leads to cosmological applications [1, 15, 18–20]; see also the upcoming book [17].

The author of the monograph under review has been very much involved, in the course of the years, with the development of this area of research. The book is intended as an introductory account that leads the readers gently from the basics to the current state of the art in the field. It is very suitable for a graduate level course covering this material (I used it myself in the past academic year for that purpose).

The book starts with a detailed discussion of the finite noncommutative geometries that are the building blocks of particle physics models. In particular, the notion of a *spectral triple*, the basic structure extending Riemannian and spin

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geometry to the noncommutative world, is introduced first in the context of finite-dimensional algebras, where the conditions on the Dirac operator and the algebra of functions can be expressed entirely in terms of linear algebra. The transition to manifold geometry and noncommutative Riemannian spin manifolds takes place in the following section and is followed by a discussion of the local index formula in noncommutative geometry. The local index formula, which is notoriously a technically difficult subject (see [12,16]) is introduced with helpful pedagogical examples, such as circles and tori, and is accompanied by a quick introduction to Hochschild and cyclic homology. The remaining sections (Part II of the book) deal specifically with particle physics models and, in particular, with gauge theories. The author first discusses the role of unitary inner symmetries and Morita equivalences in determining gauge groups and gauge potentials. The spectral action functional is then introduced and its perturbative expansion is discussed. The case of an almost-commutative geometry is then analyzed in detail in the following section. The remaining sections describe specific models: electrodynamics, Yang–Mills, the full Standard Model, ending with a section on phenomenology. The book is an excellent introduction to the field, written in a very clear, readable, and engaging way.

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