MATHEMATICAL PERSPECTIVES

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 54, Number 1, January 2017, Pages 117–122
http://dx.doi.org/10.1090/bull/1559
Article electronically published on October 5, 2016

ABOUT THE COVER: NEWTON'S *PRINCIPIA*

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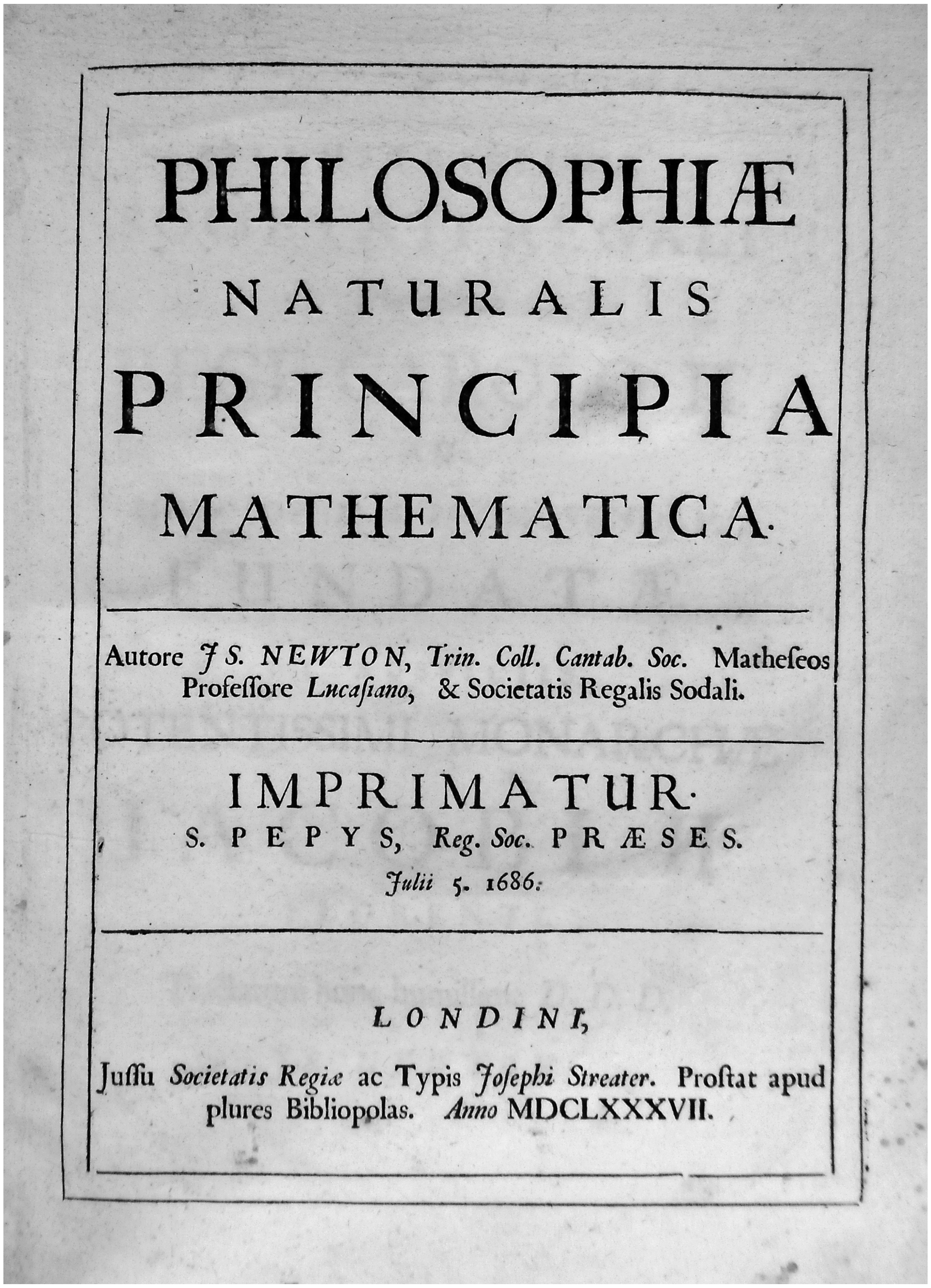
It is easy to connect the appearance of Leibniz's short article in the *Acta Eruditorum* of 1684, "Nova Methodus pro Maximis et Minimis", with Newton's famous work *Philosophiæ Naturalis Principia Mathematica* published three years later. They are both pioneering works that did in their way portend the introduction of calculus into European scientific life. There are, however, differences. First of all, one must be careful to make a distinction between differential and integral calculus. Horblit in his *One Hundred Books Famous in Science* [1] straightforwardly describes the Leibniz article succinctly as the "discovery of the differential calculus." By contrast the Newton text is described there as the "Foundation book on dynamics and gravitation. Probably the most influential scientific publication of the seventeenth century." Of course, we know that lying underneath the wordy Newton text (284 leaves!) there has to be some notion of calculus. Newton's classic was issued in three editions, each consisting of three "Books". Only Book I was available for the 1687 publication date; the other two had to be printed later by different printers. Books I and II were quite similar in format and content; the third Book, in the third edition, became the version that most influenced readers.

There is one concern to keep in mind. Notation is very, very important. For example, Newton's Second Law, $F = ma$, never appears in that form in Newton's *Principia*. He relies on the following formulation in all three editions: "A change in motion is proportional to the motive force impressed and takes place along the straight line in which that force is impressed" [5]. Is it any wonder Newton's text is usually described as difficult to read? Let us look at another example, a passage from the *Principia* [9, Sec. 10] that is often cited:

> I have not as yet been able to deduce from phenomena the reason for these properties of gravity, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a



PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.

S. PEPYS, *Reg. Soc.* PRÆSES.

Julii 5. 1686.

LONDINI,

Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

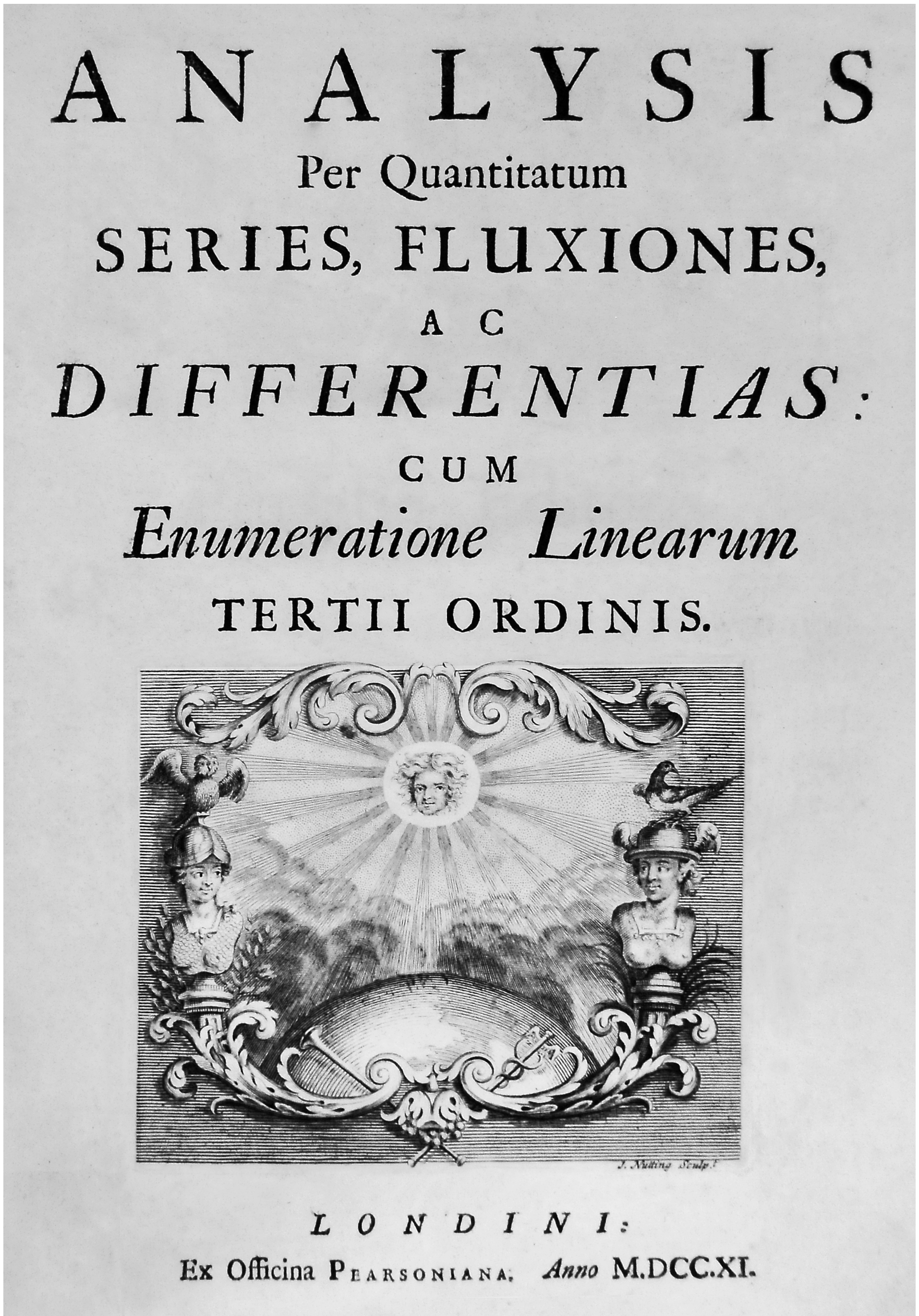
FIGURE 1. CLEAR COPY OF THE COVER. A title page of Newton's *Philosophiæ Naturalis Principia Mathematica*. (In another valid first edition the information following the city of publication is modified: Jussu Societatis Regiæ ac Typis Josephi Streater, Prostant Venales apud Sam. Smith ad insignia Principis Walliæ in Cœmitario D. Pauli, aliosq; nonnulos Bibliopolas, Anno MDCLXXXVII.)

> hypothesis, and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and law of gravity have been found by this method. And it is enough that gravity should really exist and should act according to the laws that we have set forth and should suffice for all the motions of the heavenly bodies and of our sea.

Now this is not to say that the material in Newton's masterpiece is consistently opaque or at the very least difficult. There are issues raised that are of themselves of considerable interest. He asks interesting questions: for example, "What causes the tides, and why do they vary in time as well as from place to place in the way they do?" Because Newton's answer—the gravitational action of the Sun and Moon—was merely qualitative, it left room to question whether the Moon attracts the Earth and, if so, by how strong a force. Also left open was the question of how the inertia and viscosity of the seas and the rotation of the Earth affect the tides, a question requiring a dynamic analysis of the motions of the seas in response to solar and lunar gravity [9, Sec. 9].

To find the Newton analogue of what was in Leibniz's 1684 *Acta Eruditorum* article, one has to go to Newton's short volume (also listed in Horblit) with the title *Analysis Per Quantitatum Series, Fluxiones ac Differentias* that was not published until 1711. Horblit defines it thus: "Newton's invention of differential calculus, which, though published at a later date, appeared to be simultaneous with, and independent of Leibniz."

Since 1906 we have known that the calculus was in some sense used by Archimedes maybe 2000 years before the work of Newton and Leibniz. He was slicing solids into very thin slices and then in some way "taking the limit" to find formulas for volumes. Thus there was more than a hint of progress in integral calculus there. It was many years later that there were foreshadowings of problems of looking for tangent lines to curves, the stuff of differential calculus. Here we find work of Fermat and Pascal and Huygens and Wallis and Collins, leading up to Leibniz and Newton in the 1680s. Was it Newton or Leibniz who came first? Leibniz published first with his "Nova Methodus pro Maximis et Minimis," and Newton brought out his book, tagging along behind, in 1687. It has long been assumed that some of the enormous success of mathematicians (beyond the extraordinary genius of Euler and the Bernoullis and a few others on the Continent) was due, at least in part, to the superior notation of Leibniz. It may be a more significant factor than people thought. For example, the important law named for Newton, $F = ma$, really could not be expressed by Newton in his 1687 work because he did not have notation for it. Of course, mathematics was not dead in Britain by any means, but the kind of mathematics being done there was largely the mathematics of physics and astronomy. The subject was tilted in the direction of what we would more narrowly restrict to subjects now taught in courses in physics: mechanics, planetary motion, fluid flow, among others. This was not all due to Newton's choice of notation, of course, because there was a long tradition of applications in English mathematics.



ANALYSIS
Per Quantitatum
SERIES, FLUXIONES,
AC
DIFFERENTIAS:
CUM
Enumeratione Linearum
TERTII ORDINIS.

J. Nutting Sculp.

LONDINI:
Ex Officina PEARSONIANA. *Anno* M.DCC.XI.

FIGURE 2. Title page of Newton's *Analysis Per Quantitatum Series, Fluxiones ac Differentias* [4].

There were active, creative people working in these areas in England well before Newton's time: John Wallis, for one, and others who were doing calculus problems in spite of poor notation and in spite of the obvious handicap that these posed. But there was no English Euler or the Bernoullis to follow up with an amazing flow of great contributions to a wide range of mathematical topics.

The first edition of the *Principia* got off to a rocky start. Book I was not primarily concerned with calculus. Nor was the second book. Again much of the text was devoted to planetary motion [1, No. 78]. Valid first editions have two different title pages, one reflecting corrections. The book consisted of 284 leaves and one folding plate. The title page reads: Philosophiæ Naturalis Principia Mathematica. Autore Is. Newton, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali. Imprimatur. S. Pepys, Reg. Soc. Præses. Julii 5, 1686. Londini, Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII. The second edition came along in 1713, published in Cambridge, without significant changes that would have made it more accessible. It was edited by Roger Cotes. The third edition appeared in London in 1726, one year before Newton's death, and this remained the most important edition for many years, though for the English-only population there was a delay of three years before the translation from Latin into English by Andrew Motte appeared in 1729.

FIGURE 3. A head piece of Newton's *Analysis*.

Let us close with a digression on the history and importance of the estimable journal in which Leibniz published his two ground-breaking papers on the calculus, the *Acta Eruditorum*. It was founded two years before Leibniz's 1684 paper and Leibniz was involved with its founding. It had a long life; roughly a century of publication. And what makes it remarkable is its being what its title implies: an eclectic collection of scholarly articles on a wide-ranging set of subjects. The list of mathematicians contributing papers for publication in the *Acta Eruditorum* was impressive: Euler, Jacob and Johann Bernoulli, Halley, Huygens, Laplace, L'Hôpital, Sturm, Tschirnhaus, Wolff, among many others, but Newton's name is seen rarely on tables of contents of issues of the *Acta Eruditorum*. Newton seemed to prefer publishing in journals of the Royal Society.

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