MATHEMATICAL PERSPECTIVES

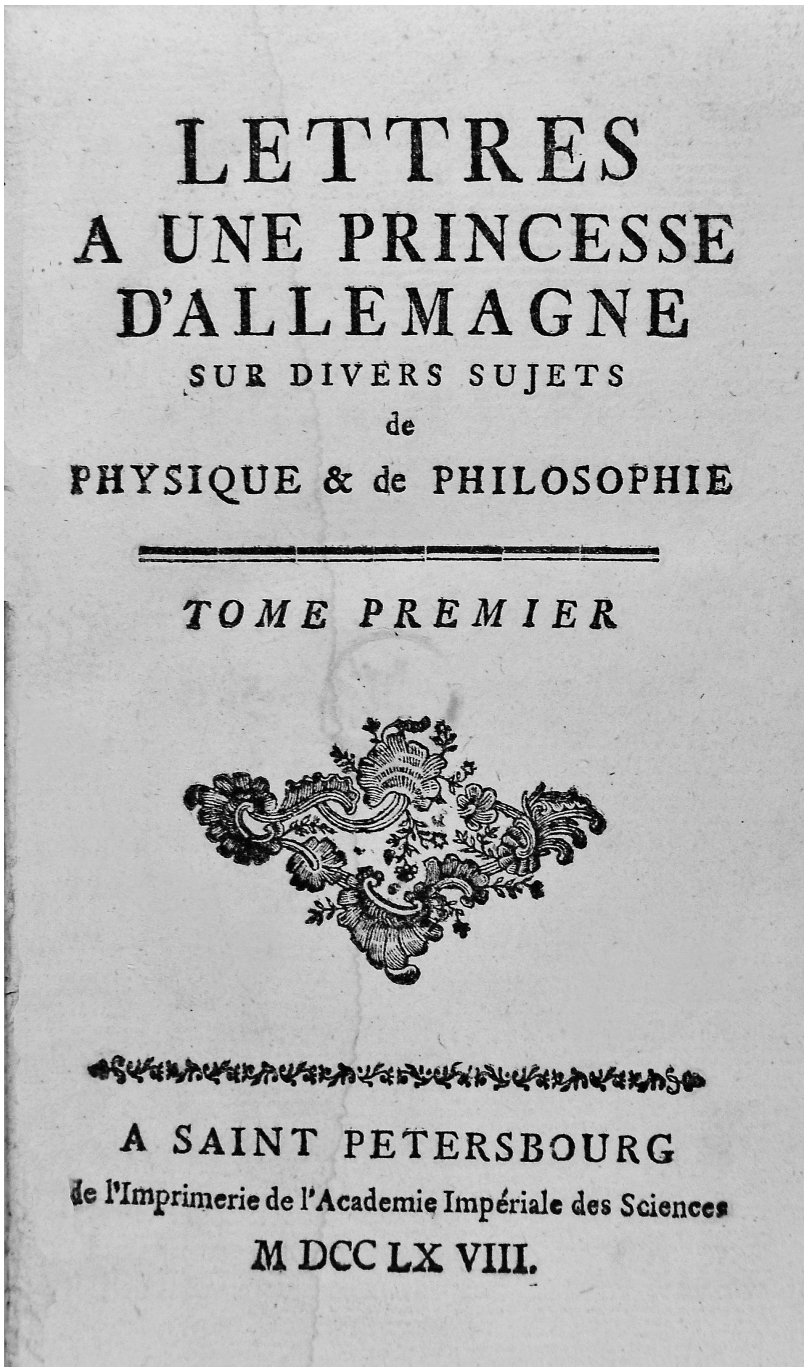
BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 54, Number 3, July 2017, Pages 499–504
http://dx.doi.org/10.1090/bull/1577
Article electronically published on May 24, 2017

ABOUT THE COVER: LEONHARD EULER, TEACHER AND MENTOR

GERALD L. ALEXANDERSON

> It was long a matter of surprize [*sic*] to me, that a Work so well known, and so justly esteemed, over the whole European Continent, ... should never have made it's [*sic*] way into our Island, in the language of the Country. While Petersburg, Berlin, Paris, nay the capital of every petty German principality, was profiting by the ingenious labour of this amiable man, and acute philosopher, the name of ... was a sound unknown to the ear of youth in the British metropolis. I was mortified to reflect that the specious and seductive productions of a *Rousseau*, and the poisonous effusions of a *Voltaire*, should be in the hands of many young men, not to say young women, to the perversion of the understanding, and the corruption of the moral principle, while the simple and useful instructions of the virtuous ... were hardly mentioned. [6, pp. xiii–xiv]

Who wrote these words? There are several clues that it was someone from the west side of the English Channel, and from the comments on Rousseau and Voltaire, we suspect that the writer was not French! He was indeed British (but not English) and the subject of his remarks was Swiss, though he worked in Russia and Germany. It is Euler, of course, and the words come from the introduction to a translation into English of Euler's *Lettres à une Princesse d'Allemagne* (see Figure 1). The translator, Henry Hunter, held a doctor of divinity degree. That helps to explain the high moral tone of his introduction. The focus on morality was no accident—Hunter was a well-known Scottish preacher and a graduate of the University of Edinburgh. He went on to translate from German a work by Euler on electricity.


LETTRES
A UNE PRINCESSE
D'ALLEMAGNE
SUR DIVERS SUJETS
de
PHYSIQUE & de PHILOSOPHIE

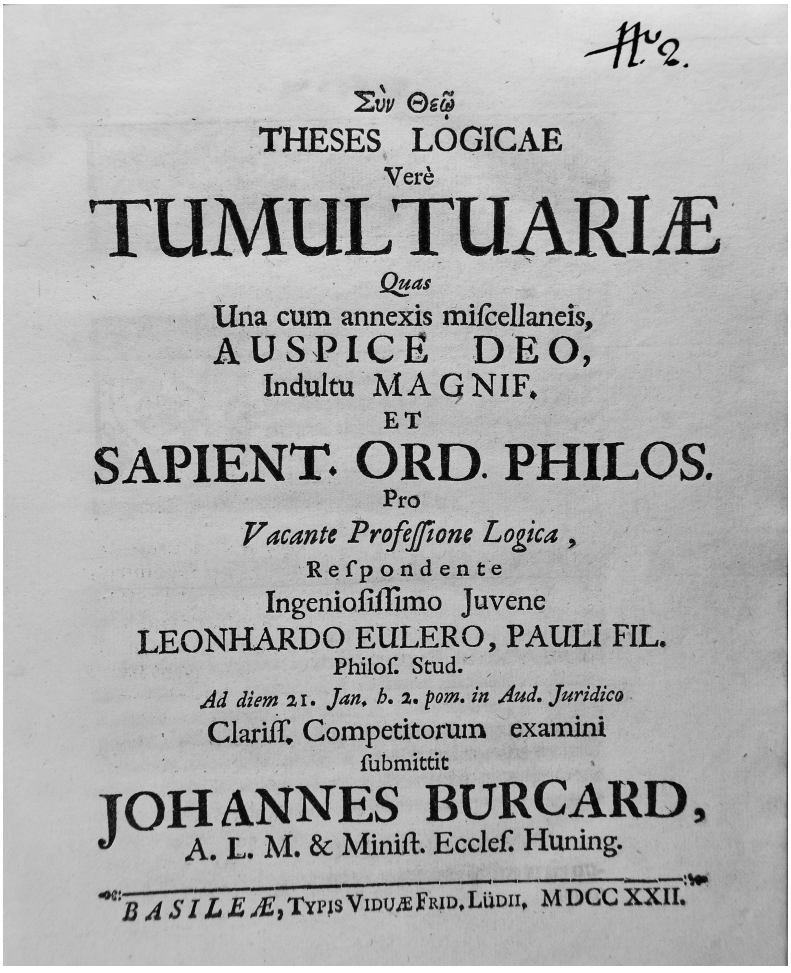
TOME PREMIER

A SAINT PETERSBOURG
de l'Imprimerie de l'Academie Impériale des Sciences
M DCC LX VIII.

FIGURE 1. ABOUT THE COVER. The title page of Euler's *Lettres à une Princesse d'Allemagne*, the first edition of 1768.

Euler was born in Basel in 1707, and when he went to the university he was initially directed to the study of theology by his father, a Swiss Reform pastor, but he soon attracted the attention of Jean Bernoulli, who recognized his enormous talent for mathematics. Euler's first two publications were responses to dissertations presented to the University of Basel, and were written when Euler was 15 ([5]; see Figure 2). These two essays are exceedingly scarce, and for years the only copies were thought to be in the library of the University of Basel. They were not included in the great catalogue of Euler's work compiled by G. Eneström [1], with an updated version available at `http://eulerarchive.maa.org/`. There are now copies even in California.

Euler's three-volume set of *Lettres* was first published in French in 1768 (volumes 1 and 2 in St. Petersburg [4]) and a third volume in 1774 in Frankfurt, all under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THESES LOGICAE
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ET
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Reſpondente
Ingenioſiſſimo Juvene
LEONHARDO EULERO, PAULI FIL.
Philoſ. Stud.
Ad diem 21. *Jan. h.* 2. *pom. in Aud. Juridico*
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ſubmittit
JOHANNES BURCARD,
A. L. M. & Miniſt. Eccleſ. Huning.
BASILEÆ, Typis Viduæ Frid. Lüdii, MDCCXXII.

FIGURE 2. The title page of Euler's first published work, a commentary on a thesis defense at the University of Basel, published in 1722 when Euler was but 15—described as an ingenious youth [5].

the title *Lettres à une princesse d'Allemagne sur divers sujets de physique et de philosophie*, written to Friederike Charlotte Brandenburg-Schwedt, also sometimes called the Princess of Prussia. The letters were written at the request of Frederick the Great, King of Prussia, who had brought Euler and his family from St. Petersburg to Berlin in 1741. Euler seemed to be popular with enlightened and not unduly modest sovereigns. Having narrowly missed working for Peter the Great in Russia, he later had as patrons Catherine the Great in Russia and Frederick the Great in Germany.

One might well assume that these expository-pedagogical letters were the work of Euler's youth when he had not yet hit his stride as a creative mathematician, or that he wrote them after his intense period of creativity was behind him and he was semiretired. That might have been the case with any ordinary person. But not for Euler! If he wrote these letters during 1760 and 1762, he was at the height of his powers. He had already written major books: his *Mechanica* (1736); the *Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes* (1744), the beginnings of the calculus of variations; his *Opuscula varii argumenti* (1746–1751); his masterpiece, the *Introductio in analysin infinitorum* (1748), in which he introduced problems that became the theory of partitions, problems on the sums of series, and all kinds of previews of work to come; and his *Institutiones calculi differentialis* (1755). Yet to appear were his *Institutionum calculi integralis* (1768–1770), his *Vollständige Anleitung zur Algebra* (1770), the *Opuscula analytica* (1783–1785), and the *Theoria motus corporum solidorum seu rigidorum* (1765), all on mathematics. And scattered throughout his life were his books on naval architecture (probably of interest to his patron), on astronomy, and his work on music, the *Tentamen Novæ Theoriæ Musicæ* of 1739. And in between all of these he turned out the three volumes of letters to the princess. The Eneström numbers are roughly chronological. The *Lettres* volumes are assigned the Eneström number 343 and his last publication was given the Eneström number 866, leaving over 500 publications yet to go. Euler was never at a loss for words, or ideas, or conjectures.

Euler's name is ever present in mathematical texts: his name is attached to numbers, constants, formulas, identities, and equations; throughout analysis—real and complex—number theory, topology, graph theory, differential equations, and on and on. And in many ways his work is the first to look modern in that he introduced notation and conventions that are still standard today.

So what was he writing about in these letters? There is not much that we would encounter in mathematics courses today, but there are generous insights on natural phenomena: the idea of magnitude, initially "the smallest as well as the greatest extension of matter actually discoverable in the system of the Universe" [3]. To teach some idea of a mile, he pointed out that the distance from [Berlin] to Magdeburg was computed to be 18 miles, and this was more clear than saying that the distance is roughly 432,000 feet. He then moved on to the diameter of the Earth, and a calculation of the distance to the Sun. (A footnote pointed out that these were German miles, not English miles, in case the reader tried to check Euler's arithmetic.)

So that is how it went as he moved from the very concrete at the beginning, stretching the imagination a bit as the story went on. Then we look at the velocity of sound, from which he moved to vibrating strings and thence to music. He devoted considerable attention to a question that is still discussed today. Why are some

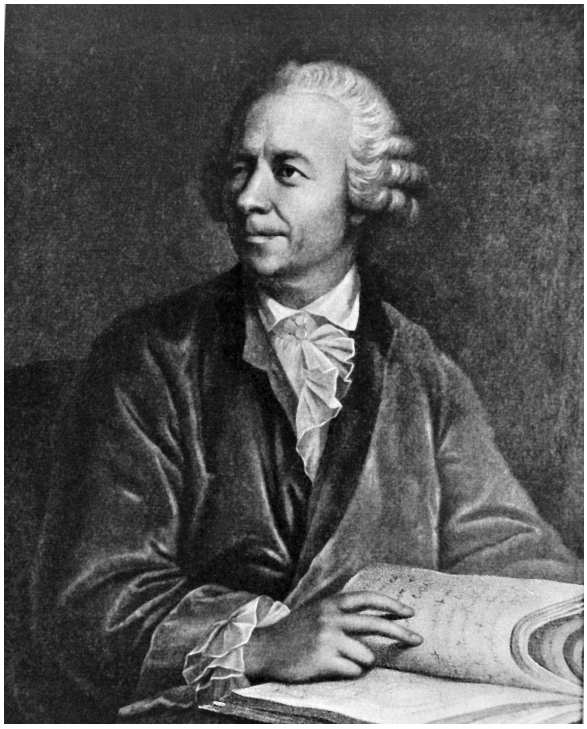

FIGURE 3. Leonhard Euler

musical intervals more consonant than others? He had discussed this earlier in the *Tentamen*. He also addressed the compression of air, elasticity of air, eventually addressing questions about the barometer. Of course, on the use of the barometer, Pascal got there first. Eventually, Euler moved to questions that we would most likely not see in science books today: the relationship between the soul and the body, for example. Here his language turned almost evangelical at times as he reminded the princess of the need to recognize God in these matters. Some of this was deleted by the translators into other languages—not surprising during that Age of Enlightenment. But he also found pages to devote to gravity (particularly the tides). And meanwhile he addressed issues on electricity, color, longitude and latitude—questions of least distances on the surface of a sphere, for example—telescopes and microscopes, magic lanterns (!), and, ultimately, very near the end, a letter "Sur le bleu du ciel", the question of why the sky is blue.

We shall not address here the effect on the princess. For many years there seemed to be little interest in the story of the princess herself. Like other minor members of the royal families in Germany (and there were so many!), she received little attention. There probably remain questions, however, that could be answered were the actual letters Euler wrote to be found. The editing of these in the various editions has been uneven and the location of the actual letters is not known. Perhaps some discoveries are yet to be made—and reported on at another time. We do know, however, that the princess herself spent much of her later life administering a convent for titled young ladies from the various noble families of Central Europe. Euler's influence on that work is not clear.

Acknowledgment

The author wishes to express gratitude to one of the ultimate sources of information about Euler, Dominic Klyve, a frequent contributor to the work of the Euler Society, for having read a draft of this note. Leonard F. Klosinski graciously provided the illustrations, which are used with permission.

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Department of Mathematics and Computer Science, Santa Clara University, 500 El Camino Real, Santa Clara, California 95053-0290
E-mail address: galexanderson@scu.edu