BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 57, Number 1, January 2020, Pages 145–151
https://doi.org/10.1090/bull/1645
Article electronically published on August 22, 2018

Fourier restriction for hypersurfaces in three dimensions and Newton polyhedra, by Isroil A. Ikromov and Detlef Müller, Annals of Mathematical Studies, No. 194, Princeton University Press, Princeton, NJ, 2016, viii+258 pp., ISBN 978-0-691-17055-8, List price US$165 (hardcover), US$75 (paperback)

The book by Ikromov and Müller is concerned with inequalities of the form

$$\Big(\int |\widehat{f}|^q d\mu \Big)^{1/q} \le C(p,q,\mu) \|f\|_p, \tag{1}$$

where μ is a finite Borel measure on $\mathbb{R}^N$, $\|f\|_p$ denotes the $L^p(\mathbb{R}^N)$ norm taken with respect to Lebesgue measure, and $\widehat{f} \equiv \mathcal{F}f$ denotes the Fourier transform of f. We assume $1 \le p \le 2$. Since for $f \in L^1(\mathbb{R}^N)$ the Fourier transform $\widehat{f}$ is bounded and continuous, the inequality is trivially satisfied with $p = 1$, and $q \le \infty$. For $p = 2$ the Fourier transform is an isomorphism on $L^2(\mathbb{R}^N)$ and nothing more can be said. The problem becomes interesting, and part of Fourier restriction theory, when μ is a singular measure, say surface measure on a submanifold of $\mathbb{R}^N$, and $1 < p < 2$. Note that the Hausdorff–Young inequality $\|\widehat{f}\|_{p'} \lesssim \|f\|_p$ does not exclude the possibility that $\widehat{f}$ is ∞ on a set of Lebesgue measure zero. Thus any Fourier restriction theorem with respect to singular measures can be interpreted as a subtle and nontrivial regularity statement about the Fourier transform.

The Fourier restriction problem for spheres. The first such restriction result is due to E.M. Stein who considered in an unpublished work from the 1960s the case where μ is surface measure on the unit sphere in $\mathbb{R}^N$, $N \ge 2$. Stein observed that inequality (1) holds with $q = 2$ for some range $1 \le p \le p_0$ with $p_0 > 1$. Stein's proof also applied to other compactly supported finite Borel measures which satisfy an inequality of the form

$$\sup_{\xi \in \mathbb{R}^N} (1 + |\xi|)^a |\widehat{\mu}(\xi)| < \infty \tag{2}$$

for some $a > 0$. For a surface measure on submanifolds of $\mathbb{R}^N$ such an inequality follows from mild curvature or *finite type* assumptions. In the case where μ is surface measure σ on the sphere, one has (2) with $a = \frac{N-1}{2}$, and no better. For any given finite Borel measure it is of great interest to determine the precise range of a for which the Fourier decay estimate (2) holds, and the precise range of p, q for which the harder Fourier restriction estimate (1) holds.

The "Holy Grail" in Fourier restriction theory is concerned with optimal Fourier restriction results for surface measure σ on the sphere (or more generally, on a compact convex hypersurface with nonvanishing Gaussian curvature). It is conjectured that

$$\mathcal{F} : L^p(\mathbb{R}^N) \to L^q(S^{N-1}, d\sigma) \tag{3}$$

holds in the range $1 \le p < \frac{2N}{N+1}$ under the additional restriction $q \le p' \frac{N-1}{N+1}$. Here $1/p + 1/p' = 1$. The adjoint operator is given by $g \mapsto \widehat{g\sigma}$ for g defined on the sphere. By the asymptotics for $\widehat{\sigma}$ one has that $\widehat{\sigma} \in L^r(\mathbb{R}^N)$ if and only if $r > \frac{2N}{N-1}$,

2010 *Mathematics Subject Classification.* Primary 42B20, 42B25, 43A25.

The reviewer was supported in part by an NSF grant.

and by a duality argument with $g \equiv 1$ this implies optimality of the p-range in the conjecture. For other necessary conditions there is an idea due to Knapp according to which one should test the inequality on functions whose Fourier transform is supported in thin parellelipipeds that fit the surface. These examples show that the stated condition on q is sharp.

For $N = 2$ the conjecture was proved by Fefferman and Stein (see [8]), but only partial progress has been made in higher dimensions. The best possible results with $q = 2$ (for which the p-range is $1 \leq p \leq \frac{2(N+1)}{N+3}$) are due to Tomas [24] and Stein [22]. This L^2 restriction theorem was established by composing the operator with its adjoint and proving an optimal $L^p(\mathbb{R}^N) \to L^{p'}(\mathbb{R}^N)$ estimate for the resulting convolution operator with convolution kernel $\widehat{\sigma}$. The Stein–Tomas result and subsequent developments had a tremendous impact on other problems in analysis, such as unique continuation, eigenfunction bounds for elliptic operators, and well-posedness of initial value problems for nonlinear evolution equations.

Sophisticated techniques have been developed by many researchers to make progress for certain ranges of $p > \frac{2(N+1)}{N+3}$, $N \geq 3$, for which we necessarily have $q < 2$. In particular one can mention milestone papers by Bourgain [3], Wolff [27], Tao [23], Bennett, Carbery, and Tao [2], Bourgain and Guth [4], and Guth [11] which also contain further important references. While most of these papers focus on extending the p-range, a recent paper by Shayya [21] made further substantial progress on the optimal exponent q in (3). As of the time of this writing (August 2018), current world records on the p-ranges can be found in preprints by Hong Wang [26] for dimension 3, and by Hickman and Rogers [12] for large dimensions.

There are other important directions in Fourier restriction theory which pose very serious challenges and have attracted much attention. Among them are the following.

- Fourier restriction for hyperbolic hypersurfaces
- Fourier restriction for manifolds of higher codimensions
- Fourier restriction for measures supported on certain fractal sets (such as Cantor type sets) satisfying uniform Fourier decay estimates
- Uniform Fourier restriction estimates with respect to affine measure
- Fourier restriction theory for hypersurfaces of finite type

We focus on the last topic, the subject matter of the book by Ikromov and Müller.

L^2 Fourier restriction theory for hypersurfaces of finite type. Let Σ be a compact hypersurface of $\mathbb{R}^N$, and let σ denote surface measure on Σ. Greenleaf [10] extended the arguments by Tomas and Stein to prove that

$$\mathcal{F} : L^p(\mathbb{R}^N) \to L^2(\Sigma, d\sigma) \tag{4}$$

for $1 \leq p \leq \frac{2(a+1)}{a+2}$, assuming the Fourier decay condition (2). This result suggests two problems. The first is to establish sharp uniform estimates of the form (2) depending on the geometry of the surface. Given such estimates for any fixed surface, the second question is whether Greenleaf's result yields an optimal range for the L^2 Fourier restriction theorem.

As a model case with a satisfactory solution for these problems, we consider the class of closed smooth *convex* hypersurfaces Σ of *finite line type*. Here it is assumed that every tangent line has finite order of contact with Σ. For this class we describe a sharp bound for the Fourier decay due to Bruna, Nagel, and Wainger [5]. Let $\delta < 1$, let $x \in \Sigma$, and let $x + T_x\Sigma$ be the affine tangent plane to Σ through x. Consider the caps

$$\mathcal{C}(x, \delta) = \{y \in \Sigma : \operatorname{dist}(y, x + T_x\Sigma) \leq \delta\}.$$

For a given ξ with $|\xi| \geq 1$, there are two points $x_\pm(\xi)$ at which ξ is normal to Σ at $x_\pm(\xi)$. Let $v_\pm(\xi)$ be the surface area of $\mathcal{C}(x_\pm(\xi), |\xi|^{-1})$. Then there is a constant C such that

$$|\widehat{\sigma}(\xi)| \leq C\big(v_+(\xi) + v_-(\xi)\big).$$

Let $a_\circ$ be the infimum over all a such that

$$\sup_{x\in\Sigma}\sup_{\delta<1} \delta^{-a}\sigma(\mathcal{C}(x, \delta))$$

is finite. Then Greenleaf's result implies that (4) holds for $1 \leq p < \frac{2(a_\circ+1)}{a_\circ+2}$. Since the caps are convex and, by John's theorem, can be compared with ellipsoids, the Knapp idea can be applied to show that this result is sharp, up to the endpoint.

The above result by Bruna, Nagel, and Wainger is uniform over large compact families of convex surfaces of finite line type. There is currently no general version of this phenomenon for nonconvex surfaces. Moreover, new phenomena arise in the nonconvex case as one has examples of hypersurfaces for which Greenleaf's theorem does not give the optimal p-range for the L^2 Fourier restriction results. The first such examples came in the form of surfaces of revolution, in a 1990 preprint by Schulz [20], which unfortunately has remained unpublished and thus largely unnoticed in the Fourier analysis community.

Surfaces in $\mathbb{R}^3$. The book by Ikromov and Müller aims for complete Fourier restriction results for surfaces in $\mathbb{R}^3$. The question about Fourier decay reduces to the problem of estimating oscillatory integrals with phase functions of two variables and to the stability of such estimates with respect to additional parameters. After localization and affine changes of variables, one can reduce to the case where the surface is given as a graph

$$y_3 = \phi(y_1, y_2), \text{ with } \phi(0, 0) = 0, \ \nabla\phi(0, 0) = 0,$$

where ϕ is defined in a small neighborhood $\mathcal{U}$ of the origin. Consider the measure μ defined by $\langle\mu, f\rangle = \int f(y_1, y_2, \phi(y_1, y_2))u(y_1, y_2)dy$, where $u \in C_c^\infty(\mathbb{R}^2)$ is supported in $\mathcal{U}$, with $u(0, 0) \neq 0$. Then the Fourier transform of μ is given by the oscillatory integral

$$\widehat{\mu}(\xi_1, \xi_2, \xi_3) = \iint e^{-2\pi i(y_1\xi_1+y_2\xi_2+\phi(y_1,y_2)\xi_3)}u(y_1, y_2)dy_1\, dy_2.$$

For large ξ one gets rapid decay by a straightforward integration by parts argument, unless we have $|\xi_1| + |\xi_2| \ll |\xi_3|$, which is the interesting parameter range.

There is a body of remarkable results on sharp decay estimates for oscillatory integrals with phase functions of two variables; in particular we mention the works by Varchenko [25], Arnold, Guseĭn-Zade, and Varchenko [1], Karpushkin [16], Magyar [17], Ikromov, Kempe, and Müller [13], Ikromov and Müller [15] and Greenblatt [9]; see also Phong and Stein [18] for related work on oscillatory integral operators. These bounds rely on methods of resolution of singularities. Karpushkin proved stability results which in turn also yield sharp estimates for the decay of Fourier transforms of surface measure of hypersurface of finite type. Here one drops the assumption of convexity and uses a weaker notion of finite type than above, namely the order of contact with any tangent plane is finite.

Finite type near $(0,0)$ means that not all coefficients in the formal Taylor expansion $\phi(y) \sim \sum c_{\alpha_1,\alpha_2} y_1^{\alpha_1} y_2^{\alpha_2}$ vanish. We now describe a result on the precise decay of the Fourier transform $\widehat{\chi\mu}$ where μ is defined via ϕ as above and $\chi \in C_c^\infty(\mathbb{R}^3)$ has sufficiently small support near the origin. Define the Newton polygon $\mathcal{N}(\phi)$ associated to $(\phi, 0)$ as the convex hull of the union of all quadrants $\mathcal{Q}_\alpha = \{(t_1, t_2) : t_1 \geq \alpha_1, t_2 \geq \alpha_2\}$ with $c_{\alpha_1,\alpha_2} \neq 0$. Define the Newton distance $d(\phi)$ as the value of $t \geq 1$ such that (t,t) belongs to the boundary of $\mathcal{N}(\phi)$. Let $\mathfrak{S}_0$ be the collection of (germs of) smooth maps Ψ defined in a neighborhood of $(0,0)$, with values in $\mathbb{R}^2$ satisfying $\Psi(0,0) = (0,0)$ and $\det D\Psi(0,0) \neq 0$. That is, $\mathfrak{S}_0$ is in correspondence with local coordinate systems fixing the origin of $\mathbb{R}^2$. We then define the height

$$h(\phi) = \sup\{d(\phi \circ \Psi) : \Psi \in \mathfrak{S}_0\}.$$

In addition, we shall also need the notion of linear height given by $h_{\text{lin}}(\phi) = \sup\{d(\phi \circ T) : T \in GL(2, \mathbb{R})\}$. Clearly, $d(\phi) \leq h_{\text{lin}}(\phi) \leq h(\phi)$.

A theorem by Karpushkin [16] implies that if ϕ is real analytic near $(0,0)$, then there is a neighborhood of the origin in $\mathbb{R}^3$ so that for all C_0^∞-functions χ supported in this neighborhood we have the estimate

$$|\widehat{\chi\mu}(\xi)| \lesssim (1 + |\xi|)^{-1/h} (\log(1 + |\xi|))^\nu,$$

with $h = h(\phi)$, and more precise statements on the exponent of the logarithmic term. Ikromov and Müller [15] proved extensions of this inequality to the smooth category. Together with Greenleaf's result, one gets an L^2 restriction estimate

$$\left(\int |\widehat{f}|^2 \chi d\mu \right)^{1/2} \lesssim \|f\|_p \tag{5}$$

in the range $1 \leq p < \frac{2h+2}{2h+1}$.

We say that our coordinate system is *adapted to* ϕ if $d(\phi) = h(\phi)$. In the case of adapted coordinates or, more generally, if $h_{\text{lin}}(\phi) = h(\phi) \equiv h$, it was shown by Ikromov and Müller [15] using arguments from Littlewood–Paley theory that (5) remains true for the endpoint $p = \frac{2h+2}{2h+1}$, and also that the resulting p-range for (5) is optimal.

The last result raises the questions on whether adapted coordinates exist and how they are constructed. Moreover, one would like to have checkable criteria under which a given coordinate system is adapted. The existence of adapted coordinates was shown by Varchenko [25] in the real analytic case, using deep results on resolution of singularities. Phong, Stein, and Sturm [19] gave a more elementary proof using Puiseux expansions. These ideas were further developed by Ikromov and Müller [14] who gave the proof of existence and a construction of adapted coordinates in the smooth category. They also provided verifiable characterizations of when a coordinate system is adapted to a given ϕ.

In order to describe such a characterization, we need some notation. Let the principal face $\pi(\phi)$ of $\mathcal{N}(\phi)$ be the face of minimal dimension containing the point $(d(\phi), d(\phi))$. The principal part of ϕ is the formal power series

$$\phi_{\text{pr}}(y_1, y_2) = \sum_{(\alpha_1,\alpha_2) \in \pi(\phi)} c_{\alpha_1,\alpha_2} y_1^{\alpha_1} y_2^{\alpha_2}.$$

Note that $\phi_{\rm pr}$ is a mixed-homogeneous polynomial when $\pi(\phi)$ is compact. In the latter case let $\mathrm{ord}_{S^1}(\phi_{\rm pr})$ be the maximal order of vanishing of $\phi_{\rm pr}$ along the unit circle S^1. Ikromov and Müller showed in [14] that ϕ is given in adapted coordinates if and only if one of the following conditions is satisfied:

(i) The principal phase $\pi(\phi)$ is a compact edge and $\mathrm{ord}_{S^1}(\phi_{\rm pr}) \le d(\phi)$.

(ii) $\pi(\phi)$ is a vertex.

(iii) $\pi(\phi)$ is an unbounded edge.

These conditions already came up in Varchenko's work, but their necessity for adaptedness is new in [14].

The case of nonadapted coordinates. The new results in the book by Ikromov and Müller deal with the case where $h_{\rm lin}(\phi)$ is strictly smaller than $h(\phi)$, i.e., the *nonadapted case*. The main theorem by Ikromov and Müller gives a sharp $L^p \to L^2$ restriction estimate for these situations under the assumption that ϕ is real analytic. The theorem identifies a quantity $H = h^{\rm restr}(\phi)$, labeled the *restriction height*, such that the $L^p \to L^2$ restriction estimate (5) holds if and only if $1 \le p \le \frac{2H+2}{2H+1}$. In the present case of nonadapted coordinates the restriction height is shown to satisfy

$$h(\phi) - 1 \le H < h(\phi)$$

so that in this situation the endpoint exponent $\frac{2H+2}{2H+1}$ is strictly larger than the exponent $\frac{2h+2}{2h+1}$ that can be obtained via Greenleaf's theorem using the uniform decay estimates for $\widehat{\chi\sigma}$.

The restriction height H can be explicitly computed from various mixed homogeneous expressions associated with edges of the Newton polygon. For the precise definition see chapters I.3 and I.4 of the monograph. To mention a simple model example, let $\phi(x_1, x_2) = (x_2 - x_1^m)^n$. Adapted coordinates are given by $(y_1, y_2) = (x_1, x_2 - x_1^m)$, and one gets $h(\phi) = n$. The algorithm for the restriction height gives $H = n - 1$ if $n \ge m + 1$ and $H = n - \frac{n}{m+1}$ if $n \le m + 1$.

The remarkable theorem by Ikromov and Müller yields a complete answer for the L^2 restriction problem for real analytic compact submanifolds of finite type in $\mathbb{R}^3$. In fact, the authors go much further as they extend this result to the smooth case under an additional factorization condition. This additional condition is shown to be satisfied in the real analytic category, and it remains open whether it is needed in general.

Future directions. The results discussed above suggest various further avenues of research. An obvious question is: What happens for hypersurfaces in higher dimensions? Sharp decay estimates for the Fourier transform of surface carried measure on finite type hypersurfaces in $\mathbb{R}^N$, $N \ge 4$, are open in general, in part due to the failure of Karpushkin's stability result in higher dimensions.

One may also ask for uniform estimates over large families of hypersurfaces in $\mathbb{R}^3$, in which a lack of curvature is compensated by damping factors incorporated in the measure. In some cases (see, e.g., [7]) such damping factors are given by suitable powers of the Gaussian curvature which leads to the consideration of affine surface measure.

Finally, given the progress on the restriction estimates in the nonvanishing curvature case beyond L^2 theory, it is natural to look into similar $L^p \to L^q(d\sigma)$ estimates for more general surfaces in $\mathbb{R}^3$. Even simple examples of finite type surfaces pose significant challenges. For recent results on such bounds, we refer the reader to a paper by Buschenhenke, Müller, and Vargas [6].

The book by Ikromov and Müller. The book under review is a research monograph that presents a new and deep result in Fourier analysis. The main theorem on restriction estimates in the case of nonadapted coordinate systems is an impressive achievement. The proof is a *tour de force* introducing several new ideas and techniques for the estimation of oscillatory integrals. The dedicated reader will need considerable amounts of time and patience to work through the details. The authors should be applauded for doing a very nice and thorough job explaining the background, the tools, and the overall strategy.

Acknowledgments

The reviewer thanks Alex Nagel and Brian Street for their helpful comments and suggestions on an earlier draft of this review.

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