

# BOOK REVIEWS

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*Philosophy of Mathematics*, by Øystein Linnebo, Princeton Foundations of Contemporary Philosophy, Princeton University Press, Princeton, NJ, 2017, vi+203 pp., US\$29.95, ISBN 978-9-691-16140-2 (English)

Philosophy of mathematics is only a little over a century old, but its roots can be traced all the way back to Plato. Descartes, Leibniz, Hume, Kant, Mill, and other philosophers discussed mathematics in interesting and insightful ways, but philosophy of mathematics only began to emerge as a distinct field in the late nineteenth and early twentieth centuries, when important developments in mathematics began to make philosophy of the subject a specialized field requiring a distinctive kind of expertise. In principle, philosophy of mathematics is a broad subject that can cover issues as varied as ontology (the character of numbers, sets, and other fundamental entities that mathematicians study); epistemology (issues concerning mathematical knowledge); the nature of mathematical practice, proof, and evidence; the application of mathematics to science and its status as a science; and so forth. In practice, philosophers of mathematics tend, quite reasonably, to focus their work more narrowly on certain issues of ontology and epistemology, and treat the other issues as deriving from these.

Øystein Linnebo's *Philosophy of Mathematics* is an excellent introduction to the subject. It has the three things one wants in such a book: a brief presentation of the key figures and their views, a display of how argumentation is practiced, and a review of different perspectives on the subject. Philosophical introductions often cover one or perhaps two of these, but Linnebo's book exhibits all three in an efficient and effective style.

Linnebo structures the book so that the reader gets the sense of being taken on a journey from the field's beginnings to its present by a seasoned and helpful guide. Realizing that you may be an amateur, the guide stops periodically at the route's key turning points, describing why that vista is important. But Linnebo also makes it clear that he is taking you on only one of several possible routes—the best route as he happens to see it—though other guides may take people on different paths that end up in different places. Still, Linnebo is a reliable guide, his route is sound, and by paying attention to what he says, you can understand why those other guides might have taken you on these different paths.

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Linnebo is refreshingly transparent in acknowledging the influence of his own spiritual guide to philosophy of mathematics: Gottlob Frege, a key founding figure of the field. Frege argued in a Platonic vein that mathematics was a science like any other, concerned with establishing facts about real objects. “Just as geographers discover continents and oceans”, Linnebo writes paraphrasing a remark by Frege himself (p. 1), “so mathematicians explore numbers and sets”.

In the journey Linnebo proceeds to describe, we look mainly at two issues: the idea that mathematical objects are real objects, as well as what kind of objects these are; and an explanation of how individual humans can come to know truths about such objects. Linnebo calls this latter issue the “integration challenge”, because it concerns how to integrate the ontology of mathematics (the objects it is about) with epistemology (ability to acquire knowledge of these eternal and abstract objects) (p. 13). Linnebo follows with an overview of Immanuel Kant’s influential conclusion that mathematical knowledge is synthetic a priori—that is, not logical, analytic, or true by definition, but requiring some additional source such as intuition. But Linnebo also points out alternate conclusions, such as that mathematical theorems are derived from experience (Mill), that the entire distinction between analytic and synthetic truths has to be revamped (Quine), or that mathematical truths are indeed both analytic and a priori, made plausible through a vast expansion of the domain of analytic knowledge (Frege)—a view called “logicism”.

Linnebo then pauses to give us a more in-depth panorama on this last view and its fate. Frege was the inventor of the *Begriffsschrift*, “the first ever formal language” that “is arguably the greatest contribution to the axiomatic method since Euclid”. Frege’s formal system vastly expanded the scope of analytic knowledge and sought to reduce arithmetic to logic. It reached its high-water mark in “Frege’s theorem”, or his proofs of all axioms of Dedekind–Peano arithmetic. Bertrand Russell then exposed its Achilles’ heel by posing the question of whether the set of all sets that are not members of itself contains itself, producing a conundrum that Frege was unable to resolve.

One attempt to move beyond the troubles Frege encountered was Hilbert’s project of effectively splitting mathematics into two parts, finitary and infinitary, in the process giving set theory, as Cantor had developed it, a solid foundation. The “one condition” that would allow this to succeed, Hilbert thought, was a “proof of consistency” (p. 69). This not only eluded Hilbert, but its possibility was refuted by Gödel’s famous incompleteness theorems. Other routes beyond Frege were the intuitionist approach charted by Brouwer and others, Mill’s empiricist approach, Quine’s attack on the entire analytic-synthetic distinction, and Benacerraf’s nominalist approach, which denies that the things with which mathematicians are concerned are objects at all.

Linnebo outlines the arguments of each thinker, along with the obstacles they encountered. Throughout the book, we meet lively characters and their thoughts, clearly expressed. These include, for instance, Hilbert’s remarks that “no one shall drive us out of the paradise which Cantor created for us”, or that removing “the principle of excluded middle from the mathematician would be the same, say, as proscribing the telescope to the astronomer or to the boxer the use of his fists” (p. 68). Vivid selections like this enliven the journey and help us amateurs look forward to what is to come. The final five chapters outline some of the most recent work in all these approaches. The book concludes with a clear and brief summary of Linnebo’s own perspective.

Linnebo is frank about the fact that he is not addressing the views of several thinkers, including Wittgenstein, who have written about mathematics; also that he is not discussing details of issues such as explanation and practice. But to my mind, this book's chief shortcoming is that Husserl does not get enough story-time. Husserl's response to the question of what sort of objects mathematicians deal with—Do they have the reality of sticks and stones? Or abstract objects?—is that this sort of question cannot be answered dogmatically but only by carefully attending to and describing how we encounter them in a practice he called “phenomenology”, which radically reconceived the character of evidence in mathematics and every other field. Yet while the other thinkers whom Linnebo discusses have their major works listed in the bibliography and at the ends of chapters, Husserl [1] does not. But outlining the phenomenological approach in introductory form is perhaps a task best carried out by a phenomenological guide.

## REFERENCES

- [1] E. Husserl, *Logical Investigations, Volumes 1 and 2*, International Library of Philosophy, Routledge, Oxon, UK, 2001.

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