## Appendix II, Part I

# Sampling and Estimation Procedures 

Rui Jiao and Bradford Chaney, Westat

## Overview

A stratified, simple random sample was employed in the CBMS 2015 survey, and strata were based on three variables: curriculum, highest degree level offered, and total institutional enrollment. Data were collected using an online survey with email and telephone followup.

## Sampling Approach

For CBMS 2015, the basic design was a stratified simple random sample of institutions. Neyman allocation based on a key outcome variable was used to determine targeted sample sizes for the 29 sampling strata. A two-phase sample design was applied to some of the strata to ease data collection workload when the sampling frame was imperfect.

## Target Population and Sampling Frames

The Integrated Postsecondary Education Data System (IPEDS), a database maintained by the National Center for Education Statistics within the U.S. Department of Education, was used as a basis for building a frame for this survey. For the academic year 2013-2014, there were approximately 3,300 four-year colleges and universities across the country and 2,600 two-year colleges, according to IPEDS. Of these, 2,501 had mathematics or statistics departments (or both). AMS conducts annual surveys of four-year institutions, and thus has reasonably current information for four-year institutions; this information was used as a basis for updating the IPEDS frame. However, it was necessary to obtain updated information on two-year institutions, partly because they are surveyed only every five years, and partly because of variations in how they are administered. Two-year institutions are sometimes centralized (with one institution having all required information, including for branch campuses) and sometimes decentralized (with each campus maintaining its own data, and there being no integrated database); the latter must be surveyed separately, so the sampling unit becomes the campus rather than the institution. Sometimes there is a mixture of centralization and decentralization at two-year
colleges; for example, an administratively independent campus might have a satellite location that is not administratively separate from the campus. The sampling unit was that level that maintained administrative data on faculty and courses. In 2010, AMS and Westat contacted all two-year institutions in the frame to include the individual campuses, but the effort of finding all of those campuses on the frame would have been significant. To reduce the operational burden of screening the entire 2-year institutions frame, a two-phase sample was applied for CBMS 2015. The 2 -year institutions formed the frame for the first phase of sampling, and then the identification of eligible campuses took place just among the sampled institutions. In the second phase, one or two campuses were selected per decentralized institution depending on the number of campuses per institution.

The target population of the CBMS 2015 survey consisted of undergraduate mathematics and statistics programs at two-year and four-year colleges and universities in the United States. Thus the frame for the CBMS 2015 survey was divided into three parts: (A) 1,395 institutions having four-year math programs, (B) 75 institutions having four-year statistics programs, and (C) 1,031 institutions having two-year math programs, for a total of 2,501 institutions having programs eligible for participation in the survey. Note that parts A and B did not necessarily consist of mutually exclusive institutions since some institutions had both four-year math programs and four-year statistics programs. However, this was not problematic since the math and statistics programs within these institutions were the targets of interest, and the departments were sampled independently.

## Sampling Strata

The three parts of the frame were each stratified using the same two variables that were used in the previous three rounds of the CBMS survey, that is, "Highest Degree Granted by the Institution" (PhD, MA or BA) and "Institutional Full Time Equivalent (FTE) Undergraduate Fall Enrollment." After an initial investigation on the population distributions of the
two variables, it was determined that the strata from CBMS 2010 largely could be maintained with a few exceptions. The stratification for part A was similar to the design in CBMS 2010 except for a change in the boundaries between strata 4 and 5 . The lower bound of stratum 5 was increased to 27,500 , determined by the lowest enrollment among the certainty institutions, and consequently the upper bound of stratum 4 was increased to 27,499 . The stratification used in CBMS 2010 for part C was applied for this round except for the addition of stratum 9 , which consists of 4-year institutions offering 2 -year math programs. The stratification for part B of the frame remained unchanged. The final stratification can be seen in the first four columns of Table 1 ahead. The four-year mathematics programs were divided into fifteen strata, the four-year statistics programs were divided into five strata, and the two-year programs were divided into nine strata.

## Allocation Process

For the CBMS 2015 survey, a stratified simple random sample of 595 institutions was drawn from parts A, B and C. For CBMS 2015, since there were only 75 institutions within part B of the frame (4-year Statistics), and since each of the five strata within part B had fewer than 25 institutions, a decision was made to sample all 75 institutions, forcing strata 16-20 to be certainty strata. The remaining 520 sampled institutions for CBMS 2015 were sampled from parts A and C of the frame. The sampling rates were adjusted based on the response rates in CBMS 2010. For the 2010 CBMS, the response rate in part C was lower than it was in part A and part B. In order to maintain the overall sample size to be at the same level of CBMS 2010, the sample size of part A was reduced and the sample size of part C was increased to yield the target sizes that are comparable among parts A, B, and C. As a result, the sample for CBMS 2015 consisted of 300 institutions sampled from part A, and 220 institutions sampled from part C. The second phase selection for part C involved drawing one or two campuses if the college was decentralized. If the institution contained five or more eligible decentralized administered campuses then two campuses were selected, otherwise, only a single campus was selected. The individual campuses were selected randomly without regard to campus size or other campus characteristics. We expected about five more campuses to be selected through the second sampling phase, making a total sample size of around 600 institutions/campuses.

In order to allocate the sample optimally to each of the 24 strata, Neyman allocation was used. This form of allocation distributes sample to the strata proportionately to the overall number of institutions on the frame belonging to each stratum, while adjusting the allocation to give more sample to those strata with greater variability (larger standard deviations) with
respect to key variables. The statistics of interest in this survey involve both the counts at the student level and the counts at the institution level. In the frame for the 2015 CBMS, the most reliable information for developing the design was the student enrollment, a count at the student level, so it was used as the key outcome variable to measure variability.

For part A, the standard deviation varied substantially, ranging from 146.59 in stratum 12 to 4855.93 in stratum 10. To smooth out this broad range of variability, and not let it dominate the sample allocation, while balancing the precision of estimates at the institution level, a modified Neyman allocation, the square root of the standard deviation of the student FTE enrollment in Fall 2013, was used to allocate the sample in strata 1 through 4, and 7 through 15 . Strata 5 and 6 were selected with certainty.

For part C, the first phase sampling rate of stratum 29 was set to be the same as the overall sampling rate, which yielded selecting 3 institutions. The certainty institutions were determined by the student FTE enrollment in Fall 2013 and they were in stratum 28. The rest of the sample was distributed through strata 21 to 27 by Neyman allocation. The variability of the key estimates was measured by the standard deviation of the student FTE in Fall 2013. Unlike the 4-year mathematics programs frame, the variability was not heavily loaded in one stratum, so use of the square root was not warranted.

The first phase sample for CBMS 2015 consisted of 300 institutions from part A (including the two certainty strata, strata 5 and 6 , of size sixteen and seven, respectively), all 75 institutions from part B, and 221 institutions from part $C$ (including the one certainty stratum, stratum 28 , of size nine), for a total of 596 institutions. See Table 1 below for details of the final allocation given in the columns labeled "Universe" (or number of institutions on the frame), "Final Sample Allocation", and "Sampling Rate". The final column of Table 1 also gives the "Raw Sampling Weights" which were adjusted for non-response after the surveys were conducted. In so doing, final sampling weights were produced, which can be used for estimation purposes.

The 221 sampled institutions for part C were contacted to obtain information on the individual campuses for the second phase sampling. One institution was found to be ineligible. Of the remaining 220 sampled institutions, 19 had decentralized administered campuses, nine of which had five or more campuses, and 10 of which had less than five campuses, yielding 297 campuses subject to the second phase sampling. Table 2 gives the distribution of the sampled institutions with different levels of campuses. The number of sampled campuses, sampling rate, and the raw sampling weights at the second phase are given in the last three columns, respectively.
TABLE 1: Phase 1 - Stratum Designations and Final Allocation for

| the CBMS 2015 Study (Program Types A, B and C) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Stratum | Program Type | Highest Degree Granted | FTE <br> Undergraduate Fall Enrollment | Universe (N) | Final Sample Allocation (n) | Sampling Rate ( $n / N$ ) | Raw Sampling Weights (N/n) |
| 1 | Four-Year Math <br> (A) | PhD | 0-7,499 | 49 | 14 | 0.29 | 3.50 |
| 2 |  |  | 7,500-14,999 | 53 | 18 | 0.34 | 2.94 |
| 3 |  |  | 15,000-19,999 | 43 | 12 | 0.28 | 3.58 |
| 4 |  |  | 20,000-27,499 | 33 | 11 | 0.33 | 3.00 |
| 5 |  |  | 27,500-34,999 | 16 | 16 | 1.00 | 1.00 |
| 6 |  |  | 35,000+ | 7 | 7 | 1.00 | 1.00 |
| 7 |  | MA | 0-6,999 | 76 | 22 | 0.29 | 3.45 |
| 8 |  |  | 7,000-10,999 | 50 | 12 | 0.24 | 4.17 |
| 9 |  |  | 11,000-14,999 | 22 | 5 | 0.23 | 4.40 |
| 10 |  |  | 15,000+ | 28 | 14 | 0.50 | 2.00 |
| 11 |  | BA | 0-999 | 185 | 18 | 0.10 | 10.28 |
| 12 |  |  | 1,000-1,499 | 186 | 16 | 0.09 | 11.63 |
| 13 |  |  | 1,500-2,499 | 298 | 36 | 0.12 | 8.28 |
| 14 |  |  | 2,500-4,999 | 223 | 42 | 0.19 | 5.31 |
| 15 |  |  | 5,000+ | 126 | 57 | 0.45 | 2.21 |
| 16 | Four-Year Statistics (B) | PhD | 0-14,999 | 16 | 16 | 1.00 | 1.00 |
| 17 |  |  | 15,000-24,999 | 18 | 18 | 1.00 | 1.00 |
| 18 |  |  | 25,000-34,999 | 17 | 17 | 1.00 | 1.00 |
| 19 |  |  | 35,000+ | 4 | 4 | 1.00 | 1.00 |
| 20 |  | MA/BA | All | 20 | 20 | 1.00 | 1.00 |
| 21 | Two-Year Schools (C) | N/A | 0-999 | 145 | 13 | 0.09 | 11.15 |
| 22 |  |  | 1,000-1,999 | 231 | 21 | 0.09 | 11.00 |
| 23 |  |  | 2,000-3,999 | 276 | 51 | 0.18 | 5.41 |
| 24 |  |  | 4,000-7,999 | 223 | 79 | 0.35 | 2.82 |
| 25 |  |  | 8,000-11,499 | 76 | 24 | 0.32 | 3.17 |
| 26 |  |  | 11,500-14,999 | 36 | 11 | 0.31 | 3.27 |
| 27 |  |  | 15,000-19,999 | 23 | 10 | 0.43 | 2.30 |
| 28 |  |  | 20,000+ | 9 | 9 | 1.00 | 1.00 |
| 29 |  |  | 4-year institution | 12 | 3 | 0.25 | 4.00 |


| TABLE 2: Phase 2 - Sampling rate per institution for the CBMS 2015 Study (Program Type C) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | \# Institutions | \# <br> Campuses per institution (Universe, N) | \# Sampled campuses per institution, (Final Sample Allocation, n) | Sampling Rate ( $\mathrm{n} / \mathrm{N}$ ) | Raw Sampling Weights ( $\mathrm{N} / \mathrm{n}$ ) |
| 201 centralized institutions |  | 201 | 1 | 1 | 1.00 | 1 |
| 19 decentralized institutions | 10 institutions (less than 5 campuses) | 3 | 2 | 1 | 0.50 | 2 |
|  |  | 4 | 3 | 1 | 0.33 | 3 |
|  |  | 3 | 4 | 1 | 0.25 | 4 |
|  | 9 institutions (above 5 campuses) | 6 | 5 | 2 | 0.40 | 2.5 |
|  |  | 1 | 6 | 2 | 0.33 | 3 |
|  |  | 1 | 7 | 2 | 0.29 | 3.5 |
|  |  | 1 | 23 | 2 | 0.09 | 11.5 |
| Total |  | 220 | 297 | 229 |  |  |

## Weighting Approach

Sampling weights that adjusted for non-responding institutions were created for weighted data analysis. To facilitate the calculation of standard errors, replicate weights were created using the stratified jackknife method. Nonresponse adjustments were also applied to each set of replicate weights.

## Sampling Weights

For parts $A$ and $B$, the raw sampling weight in table A serves as the base weight. (For part B, the sample of statistics departments, the base weight was equal to one since the departments were selected with certainty.) The raw sampling weight in the $h^{\text {th }}$ stratum was computed as $N_{h} / n_{h}$, where $N_{h}$ is the total number of institutions in the $h^{\text {th }}$ stratum and $n_{h}$ is the number of selected institutions in the hth stratum. For part C, the product of the raw sampling weights in tables $A$ and B serves as the base weight. Among the sampled institutions, a few were identified as ineligible for the following reasons:

- Institutions only offering math as part of general studies requirement but that were classified as a four-year mathematics program based on the sampling frame;
- Institutions having math courses required for some other programs but that were classified as a two-year mathematics program;
- Institutions having statistics courses required for some other programs, i.e. business school, but that were classified as a four-year statistics program;
- A duplicate institution was found.

The ineligible institutions were out-of-scope of the population of interest, so they were excluded from the weighting adjustment. The rest of the sample was classified as either responding institutions or nonresponding institutions. To remove bias from the estimates and reduce variability of the estimates, the base weights were adjusted for nonresponse. Within stratum $h$, a nonresponse adjustment factor, $f_{h}$ was calculated as

$$
f_{h}=\frac{\sum_{\text {eligible }} W_{h}}{\sum_{\text {responding }} W_{h}}
$$

where $W_{h}$ is the base weight. Small cells in a stratum with less than 10 institutions or large nonresponse adjustment exceeding 2.5 were collapsed with an adjacent cell within program type and highest degree granted. The analysis weight, $W_{h}{ }^{*}$ for any respondent in the $h^{\text {th }}$ stratum was computed as

$$
W_{h}^{*}=W_{h} f_{h} .
$$

See Tables 3, 4, 5 for the weights used in the fouryear mathematics, four-year statistics, and two-year mathematics categories, respectively. Two sets of weights were produced for two-year mathematics. One set of weights applied to all of the responding two-year institutions. Since some responding two-year institutions did not answer the course enrollment matrices, and in order to calculate variances for the course enrollments, a second set of weights was created for the subset of the responding institutions who also provided enrollment data. See tables 5 a and 5 b for the weights used in two-year mathematics non-enrollment estimates and enrollment estimates, respectively.

Table 3. Final sampling weights used in the four-year mathematics questionnaire

| Stratum <br> $(h)$ | Number of <br> completes | Number of <br> nonresponse | Number of <br> ineligibles | Response <br> rate | Base <br> weight $\left(W_{h}\right)$ | Nonresponse <br> adjusted <br> factor $\left(f_{h}\right)$ | Final weight <br> $\left(W_{h}{ }^{*}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 9 | 5 | 0 | 0.643 | 3.500 | 1.298 | 4.541 |
| 2 | 16 | 2 | 0 | 0.889 | 2.944 | 1.298 | 3.820 |
| 3 | 10 | 2 | 0 | 0.833 | 3.583 | 1.2 | 4.300 |
| 4 | 10 | 1 | 0 | 0.909 | 3.000 | 1.1 | 3.300 |
| 5 | 15 | 1 | 0 | 0.938 | 1.000 | 1.095 | 1.095 |
| 6 | 6 | 1 | 0 | 0.857 | 1.000 | 1.095 | 1.095 |
| 7 | 13 | 9 | 0 | 0.591 | 3.455 | 1.61 | 5.563 |
| 8 | 8 | 4 | 0 | 0.667 | 4.167 | 1.61 | 6.710 |
| 9 | 5 | 0 | 0 | 1.000 | 4.400 | 1 | 4.400 |
| 10 | 10 | 4 | 0 | 0.714 | 2.000 | 1.4 | 2.800 |
| 11 | 5 | 11 | 2 | 0.313 | 10.278 | 1.836 | 18.869 |
| 12 | 12 | 4 | 0 | 0.750 | 11.625 | 1.836 | 21.342 |
| 13 | 23 | 13 | 0 | 0.639 | 8.278 | 1.565 | 12.957 |
| 14 | 34 | 7 | 1 | 0.829 | 5.310 | 1.21 | $1.986-6.425^{*}$ |
| 15 | 37 | 19 | 0 | 0.661 | 2.211 | 1.514 | 3.346 |
| Total | 213 | 83 | 3 | 0.720 |  |  |  |

Table 4. Final sampling weights used in the four-year statistics questionnaire

| Stratum <br> $(h)$ | Number <br> of <br> completes | Number of <br> nonresponse | Number <br> of <br> ineligibles | Response <br> rate | Base <br> weight $\left(W_{h}\right)$ | Nonresponse <br> adjusted <br> factor $\left(f_{h}\right)$ | Final weight <br> $\left(W_{h}^{*}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 16 | 11 | 5 | 0 | 0.688 | 1.000 | 1.455 | 1.455 |
| 17 | 14 | 3 | 1 | 0.824 | 1.000 | 1.214 | 1.214 |
| 18 | 15 | 1 | 1 | 0.938 | 1.000 | 1.067 | 1.067 |
| 19 | 3 | 0 | 1 | 1.000 | 1.000 | 1 | 1.000 |
| 20 | 13 | 4 | 3 | 0.765 | 1.000 | 1.308 | 1.308 |
| Total | 56 | 13 | 6 | 0.812 |  |  |  |

Table 5a. Final sampling weights used in the two-year mathematics questionnaire, non-enrollment estimates

| Stratum <br> $(h)$ | Number <br> of <br> completes | Number of <br> nonresponse | Number <br> of <br> ineligibles | Response <br> rate | Base weight <br> $\left(W_{h}\right)$ | Nonresponse <br> adjusted <br> factor $\left(f_{h}\right)$ | Final weight <br> $\left(W_{h}{ }^{*}\right)$ |
| :---: | :---: | :---: | :---: | ---: | ---: | ---: | ---: |
| 21 | 8 | 4 | 0 | 0.667 | 11.154 | 2.059 | 22.961 |
| 22 | 8 | 13 | 0 | 0.381 | 11.000 | 2.059 | 22.645 |
| 23 | 26 | 22 | 3 | 0.542 | 5.412 | 1.846 | 9.991 |
| 24 | 45 | 32 | 2 | 0.584 | $2.823-5.646$ | 1.739 | $4.909-9.818$ |
| 25 | 13 | 11 | 0 | 0.542 | $3.167-12.667$ | 1.896 | $6.004-24.016$ |
| 26 | 9 | 3 | 0 | 0.750 | $3.273-8.182$ | 1.896 | $6.205-15.512$ |
| 27 | 3 | 9 | 0 | 0.250 | $2.300-9.200$ | 1.896 | $4.361-17.443$ |
| 28 | 8 | 7 | 0 | 0.533 | $1.000-11.500$ | 1.896 | $1.896-6.636$ |
| 29 | 0 | 1 | 2 | - | 4.000 | 1.896 |  |
| Total | 120 | 102 | 7 | 0.541 |  |  |  |

Table 5b. Final sampling weights used in the two-year mathematics questionnaire, enrollment estimates

| Stratum <br> $(h)$ | Number <br> of <br> completes | Number of <br> nonresponse | Number <br> of <br> ineligibles | Response <br> rate | Base weight <br> $\left(W_{h}\right)$ | Nonresponse <br> adjusted <br> factor $\left(f_{h}\right)$ | Final weight <br> $\left(W_{h}{ }^{*}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 8 | 4 | 0 | 0.667 | 11.154 | 2.059 | 22.961 |
| 22 | 8 | 13 | 0 | 0.381 | 11.000 | 2.059 | 22.645 |
| 23 | 24 | 24 | 3 | 0.500 | 5.412 | 2 | 10.824 |
| 24 | 41 | 36 | 2 | 0.532 | $2.823-5.646$ | 1.905 | $5.377-10.753$ |
| 25 | 11 | 13 | 0 | 0.458 | $3.167-12.667$ | 2.297 | $7.273-29.092$ |
| 26 | 7 | 5 | 0 | 0.583 | $3.273-8.182$ | 2.297 | $7.517-18.791$ |
| 27 | 2 | 10 | 0 | 0.167 | $2.300-9.200$ | 2.297 | $5.282-13.206$ |
| 28 | 7 | 8 | 0 | 0.467 | $1.000-11.500$ | 2.297 | $2.297-8.039$ |
| 29 | 0 | 1 | 2 | - |  | 4.000 | 2.297 |
| Total | 108 | 114 | 7 | 0.486 |  |  |  |

## Replicate Weights

Weighted estimates and standard errors were calculated using the replication method JKn (Jackknife method $n$, or the stratified jackknife method). The idea behind replication is to select subsamples (replicates) repeatedly from the whole sample, calculate the statistic of interest for each subsample, and then use these subsamples or replicate statistics to estimate the variance of the full-sample statistics. The JKn method divides the sample into subsamples by excluding one unit at a time.

For the CBMS, 74 replicates were created for the fouryear mathematics program and 61 replicates for the two-year mathematics programs. The replicates were designed in such a way so that on average, each replicate contains four to five sampled institutions. For the four-year statistics program, each sampled institution constituted a replicate except for those in stratum 19, resulting in 71 replicates. The same nonresponse adjustment used for the full sample was applied to each replicate.

In stratum 19, all the institutions were selected and all of them responded. These self-representing institutions were excluded from the computations involved in creating the replicate weights for non-self-representing institutions. Replicate weights associated with self-representing institutions were set equal to their full-sample weights. By handling the self-representing institutions in this manner, they were included in the population estimates but do not contribute to the resulting variance.

See Tables 6, 7, and 8 for the replicates for the fouryear mathematics, four-year statistics, and two-year mathematics categories, respectively.

For variance estimation purposes, the "Stratum" in Tables 6, 7, and 8 is referred as the variance stratum (VarStrat). The sampled institutions in a VarStrat are the variance units (VarUnits). For the first replicate weight, the full sample of institutions in the first VarStrat and VarUnit were multiplied by 0 and the
weights associated with the other VarUnits in the same VarStrat and adjusted by $n_{h}^{\prime} /\left(n_{h}^{\prime}-1\right)$ to account for reducing the sample. The weights of the institutions in other VarStrat were not changed. The remaining replicates were formed in the same manner by systematically dropping each of the remaining VarUnits and computing the replicate weights as described for the first replicate.

## Variance Estimation

Suppose that $\hat{\theta}$ is the full-sample estimate of some population parameter $\theta$. The variance estimator using the $J K n$ method, $v(\hat{\theta})$ is

$$
v(\hat{\theta})=\sum_{g=1}^{G}=f_{g} h_{g}\left(\hat{\theta}_{(g)}-\theta\right)^{2} .
$$

where
$\hat{\theta}_{(g)}$ is the estimate of $\theta$ based on the observations included in the $g$-th replicate,
$G$ is the number of replicates formed,
$f_{g}$ is the finite population correction (FPC) factors for replicate $g$, and
$h_{g}$ is the JKn factors for replicate $g$.
The FPC is an adjustment to the estimated variance that accounts for how large a fraction of the population is selection for the sample. For replicate $g$, the FPC factor is $f_{g}=1-m_{h} / N_{h}{ }^{\prime}$, where $m_{h}$ is the number of completes and $N_{h}$, is the total number of eligible institutions in the $h^{\text {th }}$ stratum. For the two-year mathematics, the FPC factor was calculated for the first phase of selection. The JKn factor is computed as $h_{g}$ $=\left(n_{h}{ }^{\prime}-1\right) / n_{h}{ }^{\prime}$, where $n_{h}{ }^{\prime}$ is the number of selected eligible institutions in the $\mathrm{h}^{\text {th }}$ stratum.

See Tables 6, 7, and 8 for the JKn factors and FPC factors for the four-year mathematics, four-year statistics, and two-year mathematics categories, respectively.

Table 6. Replicates, JKn factors, and FPC factors for the four-year mathematics program

| Stratum <br> (h) | Replicate <br> (g) | Number of replicates | JKn factors | FPC factors |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1-3 | 3 | 0.667 | 0.816 |
| 2 | 4-7 | 4 | 0.750 | 0.698 |
| 3 | 8-10 | 3 | 0.667 | 0.767 |
| 4 | 11-12 | 2 | 0.500 | 0.697 |
| 5 | 13-16 | 4 | 0.750 | 0.063 |
| 6 | 17-23 | 7 | 0.857 | 0.143 |
| 7 | 24-28 | 5 | 0.800 | 0.829 |
| 8 | 29-31 | 3 | 0.667 | 0.840 |
| 9 | 32-34 | 3 | 0.667 | 0.773 |
| 10 | 35-37 | 3 | 0.667 | 0.643 |
| 11 | 38-41 | 4 | 0.750 | 0.973 |
| 12 | 42-45 | 4 | 0.750 | 0.935 |
| 13 | 46-53 | 8 | 0.875 | 0.923 |
| 14 | 54-62 | 9 | 0.889 | 0.847 |
| 15 | 63-74 | 12 | 0.917 | 0.704 |

Table 7. Replicates, JKn factors, and FPC factors for the four-year statistics program

| Stratum <br> $(h)$ | Replicate <br> $(g)$ | Number of <br> replicates | JKn <br> factors | FPC <br> factors |
| :---: | :---: | :---: | :---: | :---: |
| 16 | $1-16$ | 16 | 0.938 | 0.313 |
| 17 | $17-34$ | 18 | 0.944 | 0.176 |
| 18 | $35-51$ | 17 | 0.941 | 0.063 |
| 19 | $52-55$ | 4 | - | - |
| 20 | $56-75$ | 20 | 0.950 | 0.235 |

Table 8a. Replicates, JKn factors, and FPC factors for the two-year statistics program, nonenrollment estimates

| Stratum <br> $(h)$ | Replicate <br> $(g)$ | Number of <br> replicates | JKn <br> factors | FPC <br> factors |
| :---: | :---: | :---: | :---: | :---: |
| 21 | $1-4$ | 4 | 0.750 | 0.917 |
| 22 | $5-8$ | 4 | 0.750 | 0.909 |
| 23 | $9-21$ | 13 | 0.923 | 0.815 |
| 24 | $22-38$ | 17 | 0.941 | 0.646 |
| 25 | $39-46$ | 8 | 0.875 | 0.684 |
| 26 | $47-49$ | 3 | 0.667 | 0.694 |
| 27 | $50-52$ | 3 | 0.667 | 0.565 |
| 28 | $53-58$ | 6 | 0.833 | 0.857 |
| 29 | $59-61$ | 3 | 0.667 | 0.750 |

Table 8b. Replicates, JKn factors, and FPC factors for the two-year statistics program, enrollment estimates

| Stratum <br> $(h)$ | Replicate <br> $(g)$ | Number of <br> replicates | JKn <br> factors | FPC <br> factors |
| :---: | :---: | :---: | :---: | :---: |
| 21 | $1-4$ | 4 | 0.750 | 0.917 |
| 22 | $5-8$ | 4 | 0.750 | 0.909 |
| 23 | $9-21$ | 13 | 0.923 | 0.815 |
| 24 | $22-38$ | 17 | 0.941 | 0.646 |
| 25 | $39-46$ | 8 | 0.875 | 0.684 |
| 26 | $47-49$ | 3 | 0.667 | 0.694 |
| 27 | $50-52$ | 3 | 0.458 | 0.565 |
| 28 | $53-58$ | 6 | 0.833 | 0.875 |
| 29 | $59-61$ | 3 | 0.667 | 0.750 |

WesVar, a variance estimation software designed for complex surveys, was used to calculate estimates and standard errors of the estimates for the CBMS using the JKn replication method. WesVar can be used with a wide range of complex sample designs, including multistage, stratified, and unequal proba-
bility samples. The replicate variance estimates can reflect many types of estimation schemes, including nonresponse adjustment, poststratification, raking, and ratio estimation. It computes variance estimates for medians, percentiles, ratios, difference of ratios, and log-odds ratios.

## Appendix II, Part II

# Sampling and Estimation Procedures: Four-Year Mathematics and Statistics Faculty Profile 

James W. Maxwell,<br>American Mathematical Society

## Overview

In CBMS surveys prior to 2005, information on the faculty was based on data collected on the CBMS survey form. Starting with the 2010 CBMS survey the information on the faculty at four-year colleges and universities is based on a separate survey conducted by the American Mathematical Society. The Departmental Profile Survey is one of several surveys of mathematical sciences departments at four-year institutions conducted annually as part of the AMS-ASA-IMS-MAA-SIAM Annual Survey of the Mathematical Sciences. For 2015 the Departmental Profile Survey form was expanded to gather data on the age and the race/ethnicity of the faculty, in addition to the data collected annually on rank, tenure status and gender. The information on the four-year mathematics and statistics faculty derived from this data appears in Chapters 1 and 4 of this report.

## Target Populations and Survey Approach

The procedures used to conduct the 2015 Departmental Profile survey are parallel to those used in CBMS 2015 as described in detail in Part I of this appendix. The primary characteristics used to stratify the departments for survey and reporting purposes are program type (four-year mathematics or four-
year statistics) and the highest mathematical sciences degree offered by the department: doctoral, masters, or bachelors. The 2015 Departmental Profile survey employed a census of the mathematics and statistics departments in the sample frame whereas the CBMS survey sampled these departments. In addition, the CBMS 2015 sample frame of statistics departments included sixteen departments that offered at most a masters degree in statistics. These departments are not part of the regular Annual Survey sample frame but were included in the 2015 Departmental Profile survey. The Annual Survey reports separately on doctorate-granting departments of applied mathematics, but these departments were grouped with the doctoral departments of mathematics for the CBMS 2015 analysis.

## Comparison of the Annual Survey Sample Frame with the CBMS Sample Frame

Table AS. 1 demonstrates that the sample frames of four-year mathematics and statistics departments used in the two surveys closely align. As a consequence of this alignment, the distinction between the terms "Bachelors", "Masters" and "Doctoral" Mathematics Departments as defined in the two surveys is immaterial.

Table AS. 1 Comparability of 2015 Annual Survey Sample Frame and the 2015 CBMS Sample Frame for Four-Year Mathematics Departments \& Statistics Departments

| Dept. Grouping | Annual Survey Count | CBMS Count | Overlap Count |
| :--- | :---: | :---: | :---: |
| Doctoral Math. Depts. | 201 | 201 | 200 |
| Masters Math. Depts. | 175 | 176 | 174 |
| Bachelors Math. Depts. | 1011 | 1018 | 1010 |
| Doctoral Stat. Depts. | 54 | 55 | 54 |
| Masters Stat. Depts. | 16 | 20 | 16 |
| Total |  | 1457 | 1470 |

Table AS. 2 describes the stratifications used with the the 2015 Departmental Profile data. This is the same stratification scheme used for CBMS 2015 data and described in more detail in Part I of this appendix.

## Survey Implementation

Departments of mathematics and statistics received the Departmental Profile forms in early January of 2016 asking them to report on their fall-term 2015 faculty. Non-responding departments received follow-up requests over the winter and early spring of 2016. The final effort to obtain responses took place during April, and these efforts were concentrated on the strata with the lowest response rates.

## Data Analysis

The analysis used with the 2015 Departmental Profile data parallels that used for CBMS 2015 data.

Table AS. 2 lists the final sample weights used to produce the estimates within each stratum of the counts of faculty by rank, type-of-appointment and gender. The column "Response rate" reflects the sum of the usable forms returned. The sample weights used to produce estimates of age distribution and race/ethnicity distributions were somewhat higher due to item non-response for these data. By way of comparison, Table AS. 3 shows response rates for the age data collected.

The standard errors reported for the faculty data were computed using the formulas described on pages 83-84 and 97-98 of [SMO].

Table AS. 2 Stratum designations and allocations and nonresponse-adjusted sample weights used with Annual Survey Data of faculty counts by rank, type-ofappointment and gender for the CBMS 2015 report.

| Stratum | Program Type | Highest <br> Degree | Universe <br> ( N ) | Number selected <br> ( n ) | Number of Responses | Response rate | Final sampling weights |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4-year <br> Math | PhD | 49 | 49 | 37 | 0.755 | 1.324 |
| 2 |  |  | 53 | 53 | 45 | 0.849 | 1.178 |
| 3 |  |  | 43 | 43 | 35 | 0.814 | 1.229 |
| 4 |  |  | 32 | 32 | 23 | 0.719 | 1.391 |
| 5 |  |  | 16 | 16 | 13 | 0.813 | 1.231 |
| 6 |  |  | 7 | 7 | 7 | 1.000 | 1.000 |
| 7 |  | MA | 76 | 76 | 27 | 0.355 | 2.815 |
| 8 |  |  | 50 | 50 | 31 | 0.620 | 1.613 |
| 9 |  |  | 22 | 22 | 15 | 0.682 | 1.467 |
| 10 |  |  | 28 | 28 | 11 | 0.393 | 2.545 |
| 11 |  | BA | 180 | 180 | 48 | 0.267 | 3.750 |
| 12 |  |  | 186 | 186 | 59 | 0.317 | 3.153 |
| 13 |  |  | 297 | 297 | 110 | 0.370 | 2.700 |
| 14 |  |  | 222 | 222 | 80 | 0.360 | 2.775 |
| 15 |  |  | 123 | 123 | 53 | 0.431 | 2.321 |
| 16 | 4-year Stat | PhD | 16 | 16 | 10 | 0.625 | 1.600 |
| 17 |  |  | 18 | 18 | 15 | 0.833 | 1.200 |
| 18 |  |  | 16 | 16 | 11 | 0.688 | 1.455 |
| 19 |  |  | 4 | 4 | 3 | 0.750 | 1.333 |
| 20 |  | MA | 16 | 16 | 6 | 0.375 | 2.667 |

Table AS. 3 Stratum designations and allocations and nonresponse-adjusted sample weights used with Annual Survey Data of faculty counts by age bins for the CBMS 2015 report.

| Stratum | Program Type | Highest <br> Degree | Universe <br> (N) | Number selected ( n ) | Number of Responses | Response rate | Final sampling weights |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4-year <br> Math | PhD | 49 | 49 | 28 | 0.571 | 1.750 |
| 2 |  |  | 53 | 53 | 38 | 0.717 | 1.395 |
| 3 |  |  | 43 | 43 | 26 | 0.605 | 1.654 |
| 4 |  |  | 32 | 32 | 22 | 0.688 | 1.455 |
| 5 |  |  | 16 | 16 | 13 | 0.813 | 1.231 |
| 6 |  |  | 7 | 7 | 7 | 1.000 | 1.000 |
| 7 |  | MA | 76 | 76 | 22 | 0.289 | 3.455 |
| 8 |  |  | 50 | 50 | 29 | 0.580 | 1.724 |
| 9 |  |  | 22 | 22 | 12 | 0.545 | 1.833 |
| 10 |  |  | 28 | 28 | 8 | 0.286 | 3.500 |
| 11 |  | BA | 180 | 180 | 48 | 0.267 | 3.750 |
| 12 |  |  | 186 | 186 | 57 | 0.306 | 3.263 |
| 13 |  |  | 297 | 297 | 98 | 0.330 | 3.031 |
| 14 |  |  | 222 | 222 | 74 | 0.333 | 3.000 |
| 15 |  |  | 123 | 123 | 44 | 0.358 | 2.795 |
| 16 | 4-year Stat | PhD | 16 | 16 | 7 | 0.438 | 2.286 |
| 17 |  |  | 18 | 18 | 14 | 0.778 | 1.286 |
| 18 |  |  | 16 | 16 | 10 | 0.625 | 1.600 |
| 19 |  |  | 4 | 4 | 2 | 0.500 | 2.000 |
| 20 |  | MA | 16 | 16 | 6 | 0.375 | 2.667 |

