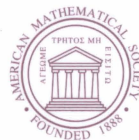

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Markov Random Fields and their Applications

Ross Kindermann
J. Laurie Snell



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PREFACE

Markov random fields is a new branch of probability theory that promises to be important both in the theory and application of probability. The existing literature on the subject is quite technical and often only understandable to the expert. This paper is an attempt to present the basic ideas of the subject and its application to a wider audience. We have relied on examples and computer graphics to convey the meaning of the results when they are too technical to prove. This graphical work was made possible by a grant to Dartmouth College from the Sloan Foundation. This work is also part of a project to produce modules on applications of probability under a grant from the National Science Foundation. The authors wish also to acknowledge the support of Kiewit Computation Center at Dartmouth College.

We were assisted by Robert Beck and William Myers in the writing of computer programs. We are very grateful to Marie Slack for a fine job of typing the material in a form suitable for a sometimes friendly and sometimes less-than-friendly computer.

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