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Umbral Calculus and Hopf Algebras

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Volume 6

Umbral Calculus and Hopf Algebras

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**PROCEEDINGS OF THE CONFERENCE ON
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PREFACE

The present volume represents a unique blending of two fields only recently recognized as related. On one hand lies the field of Combinatorics with roots (at least immediately traceable via generating functions to Umbral Calculus, the specialty at hand) in the 19th century writings of Boole on operator calculus. Both the foundations and much of the history of the Umbral Calculus are explored in great clarity in [R-K-O and R-R] which have extensive bibliographies. On the other hand is the field of Hopf Algebras, which is usually traced to the paper of Milnor and Moore [M-M] but whose first general exposition is little more than a decade old [S].

For some years, Gian-Carla Rota wrote that this theory should be directly applicable to Combinatorics, especially the Umbral Calculus, but the first distinct attention given by specialists is probably the lecture Moss Sweedler gave at the 13th Dennison Algebra Conference, Dennison College, 1978. This did not really appear in print, nor did Sweedler's Hopf Algebra colleagues seize the subject and carry it further forward.

Both Rota and Sweedler, therefore, were pleased when the University of Oklahoma was able to support their joint appearance at a conference funded by the J. C. Karcher Foundation in May, 1978. The conference centered on lectures they gave, with S. A. Joni assisting Rota. Sweedler lectured first on elementary coalgebra theory aimed at combinatorists, Rota on elementary combinatorics aimed at the algebraists. Both lectures converged toward those who were or would work at the intersection. Sweedler and Warren Nichols prepared notes of Sweedler's talks and Joni and Rota of Rota's and a mimeographed version was circulated by the Oklahoma Mathematics Department. The present volume represents an attempt to make these more accessible.

The Sweedler notes here are essentially unchanged from those distributed by Oklahoma. They aim, in a direct and elementary way, to give the reader sufficient knowledge of coalgebra theory to understand the coalgebra formulation of special sequences of polynomials.

The Rota notes are reproduced with permission from [J-R], and represent a reworking of the original, with corrections and a few additions. They contain detailed applications not only to Umbral Calculus, but to partition studies, incidence algebras, lattice theory, and other traditional spheres of combinatoric interest. The notes form a broad survey for anyone who would like detailed and concrete examples of the areas already known to be amenable to a coalgebraic approach.

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