CONTEMPORARY MATHEMATICS

Ordered Fields and Real Algebraic Geometry

AMERICAN MATHEMATICAL SOCIETY

VOLUME 8

http://dx.doi.org/10.1090/conm/008

CONTEMPORARY MATHEMATICS

Titles in this Series

VOLUME 1 **Markov random fields and their applications**
Ross Kindermann and J. Laurie Snell

VOLUME 2 **Proceedings of the conference on integration, topology, and geometry in linear spaces**
William H. Graves, Editor

VOLUME 3 **The closed graph and P-closed graph properties in general topology**
T. R. Hamlett and L. L. Herrington

VOLUME 4 **Problems of elastic stability and vibrations**
Vadim Komkov, Editor

VOLUME 5 **Rational constructions of modules for simple Lie algebras**
George B. Seligman

VOLUME 6 **Umbral calculus and Hopf algebras**
Robert Morris, Editor

VOLUME 7 **Complex contour integral representation of cardinal spline functions**
Walter Schempp

VOLUME 8 **Ordered fields and real algebraic geometry**
D. W. Dubois and T. Recio, Editors

VOLUME 9 **Papers in algebra, analysis and statistics**
R. Lidl, Editor

CONTEMPORARY MATHEMATICS

Volume 8

Ordered Fields and Real Algebraic Geometry

AMERICAN MATHEMATICAL SOCIETY
Providence • Rhode Island

PROCEEDINGS OF THE SPECIAL SESSION ON ORDERED FIELDS AND REAL ALGEBRAIC GEOMETRY 87TH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY

HELD IN SAN FRANCISCO, CALIFORNIA

JANUARY 7–11, 1981

EDITED BY

D. W. DUBOIS AND T. RECIO

Library of Congress Cataloging in Publication Data

Special Session on Ordered Fields and Real Algebraic Geometry (1981: San Francisco, Calif.)
Ordered fields and real algebraic geometry.
(Contemporary mathematics, ISSN 0271–4132; v. 8)
"Proceedings of the Special Session on Ordered Fields and Real Algebraic Geometry, 87th Annual Meeting of the American Mathematical Society, held at San Francisco, California, January 7–11, 1982"–T.p. verso.
Bibliography: p.
1. Ordered fields–Congresses. 2. Geometry, Algebraic–Congresses. I. Dubois, D. W. (Donald Ward), 1923– . II. Recio, T. (Tómas), 1949– . III. American Mathematical Society. Meeting (87th: 1981: San Francisco, Calif.) IV. Title. V. Series: Contemporary mathematics (American Mathematical Society); v. 8.
QA247.S67 1981 512'.3 82-3951
ISBN 0-8218-5007-5 AACR2

1980 Mathematics Subject Classification.
Primary 12D15, 12J15, 32C05, 58A07, 10C04, 13J25, 06F25, 03B52.
Copyright © 1982 by the American Mathematical Society
Printed in the United States of America
All rights reserved except those granted to the United States Government
This book may not be reproduced in any form without the permission of the publishers.

CONTENTS

Introduction vii

Normal decompositions of semi-algebraic sets 1
C. ANDRADAS

Some open problems 19
G. BRUMFIEL

La topologie du spectre réel 27
MICHEL COSTE AND MARIE-FRANÇOISE ROY

Semialgebraic topology over a real closed field 61
H. DELFS AND M. KNEBUSCH

A finiteness theorem for open semi-algebraic sets, with applications to Hilbert's 17th problem 79
C. DELZELL

An (almost) trivial local-global principle for the representation of -1 as a sum of squares in an arbitrary commutative ring 99
L. BRÖCKER, ANDREAS DRESS AND R. SCHARLAU

The Nash ring of a real surface 107
G. EFROYMSON

Extension of an order to a simple transcendental extension 113
R. GILMER

Théorie des modèles et fonctions définies positives sur les variétés algébriques réelles 119
D. GONDARD

Chains of higher level orderings 141
J. HARMAN

Some properties of positive derivations on f-rings 175
M. HENRIKSEN AND F. SMITH

Quadratic forms over fields with finitely many orderings 185
J. MERZEL

Finitely generated groups of divisibility 231
B. GLASTAD AND J. MOTT

On quadratic forms and Abelian varieties over function fields 249
A. PFISTER

Order extensions and real algebraic geometry 265
D. DUBOIS AND T. RECIO

Real points and real places 289
H. SCHÜLTING

The strong topology on real algebraic varieties 297
N. SCHWARTZ

The square class invariant for Pythagorean fields 327
D. SHAPIRO AND T. LAM

Coherent algebraic sheaves in real algebraic geometry 341
A. TOGNOLI

Digging holes in algebraic closures à la Artin—II 351
A. ENGLER AND T. VISWANATHAN

INTRODUCTION

The contents of this book comprise, with only minor editorial changes, papers submitted by those mathematicians who accepted our invitation to speak at the American Mathematical Society's 1981 Special Session on Ordered Fields and Real Algebraic Geometry in San Francisco. We wish to thank all those who made possible the Special Session and this publication. Special thanks are offered to Professors Kenneth Ross and James Milgram, and to our incredibly patient typist, Rose Henry.

D. W. Dubois
T. Recio