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Plane Ellipticity and Related Problems

AMERICAN MATHEMATICAL SOCIETY

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CONTEMPORARY MATHEMATICS

Volume 11

Plane Ellipticity and Related Problems

AMERICAN MATHEMATICAL SOCIETY

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**PROCEEDINGS OF THE SPECIAL SESSION ON
ELLIPTIC SYSTEMS IN THE PLANE
87TH ANNUAL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY
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INTRODUCTION

In this collection of papers concepts associated with plane ellipticity is extended in several ways. For example, the investigations of Begehr and Gilbert, Begehr and Hsiao, Hile, and Snyder treat systems of elliptic partial differential equations in the plane which resemble in some sense the Cauchy-Riemann equations. Their point of view is to seek general representation formulas and to use these in some cases to solve boundary value problems. Continuing with the theme of generalizing the Cauchy-Riemann equations Buchanan treats the Bers-Vekua type systems in two complex variables, while Delanghe and Sommen, Brackx and Pincket, and Lounesto investigate hypercomplex function theory in \mathbb{R}^n , that is the class of monogenic functions having values in a Clifford algebra.

The remaining talks comprising this special meeting cannot be categorized as falling into a general group, but rather explore isolated, albeit important topics associated with ellipticity. In the collection we have the paper by Treves which answers a fundamental question posed by Lee Rubel, namely does there exist a real homogeneous, linear partial differential equation in a domain of \mathbb{R}^2 (\mathbb{R}^n) having only the trivial solution as a solution. Another work in this general group was a paper by Protter, who showed that Payne's method of obtaining gradient bounds by means of the maximum principle may be extended to a fairly general class of semilinear elliptic equations.

The manuscripts by Bruch and Sloss, and Hummel are involved with the method of variational inequalities for problems of planar fluid flow. Hummel investigates these flows by means of the hodograph transformation, whereas Bruch and Sloss treat a free-boundary problem by first introducing the Baiocchi transformation.

Finally we have the papers by McCoy and Aziz, Door and Kellogg. McCoy uses integral operators to generate a family of basis functions in order to obtain an analogue of the Favard-Achieser-Krein approximation theorem. The paper by Aziz et. al. develops the finite element method for an exterior boundary value problem associated with a system of elliptic partial differential equations which occur in the scattering of electromagnetic waves.

Robert P. Gilbert

