

CONTEMPORARY MATHEMATICS

22

Second Edition

Factorizations of $b^n \pm 1$ $b=2, 3, 5, 6, 7, 10, 11, 12$ Up to High Powers

John Brillhart
D. H. Lehmer
J. L. Selfridge
Bryant Tuckerman
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This work is dedicated to the memory
of Lt.-Col. Allan J. C. Cunningham
(1842-1928), an industrious computer
and maker of mathematical tables.

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