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Value Distribution Theory and its Applications

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CONTEMPORARY MATHEMATICS

Volume 25

Value Distribution Theory and its Applications

Chung-Chun Yang, Editor

AMERICAN MATHEMATICAL SOCIETY
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PREFACE

Recently there have appeared numerous articles on the beautiful and classical Nevanlinna value distribution theory and its applications. More specifically, in areas such as defect relations, growth estimation of solutions of differential equations, distribution of zeros and poles of meromorphic functions, entire functions of bounded index, factorization theory, etc. These results have appeared in many different journals in various parts of the world.

In April of 1983 a special session on value distribution theory and its application was held in New York City as part of the 803rd meeting of the American Mathematical Society. The purpose of this session was to bring together some of the mathematicians working in this active field, to present the results of their research, to explore and exchange problems among themselves and to stimulate further research in this area.

Most of the papers in this collection were presented at the special session and have never before been published. For instance, announced here by Charles Osgood is his complete proof of the validity of the analogous Nevanlinna second fundamental theorem for small functions (i.e. defect relations for small functions). This has been a challenge problem for complex analysts for a long time.

I wish to thank all the mathematicians who participated in the special session and for their contributions to this volume. Special thanks are extended to those from abroad: Professors Chi-Tai Chuang and Yong-Xing Gu from China, Professors Kiyoshi Niino and Seiki Mori from Japan, Professor Shlomo Strelitz from Israel, Professor Sakari Toppila from Finland, and Professor L.S.O. Liverpool from Nigeria. Also I wish to thank the American Mathematical Society and the editorial board of the Contemporary Mathematics Series for their approval of this publication and also the editorial department for many helpful suggestions.

In conclusion, it is the unanimous feeling of all the contributors to this volume that the late Professor Rolf Nevanlinna has inspired us in many different ways in developing and searching for further results in this field, both in one variable and in several variables. Therefore it is a privilege and our opportunity to acknowledge our debt to him and to dedicate this collection to his memory.

CHUNG-CHUN YANG
WASHINGTON, D.C.

DEDICATED TO THE MEMORY OF ROLF NEVANLINNA
WHO CREATED THE BEAUTIFUL AND RICH FIELD OF VALUE DISTRIBUTION
THEORY OF MEROMORPHIC FUNCTIONS

CONTRIBUTORS TO THIS VOLUME

Chi-Tai Chuang
The Department of Mathematics, Peking University, Beijing, China

Matts R. Essén
The Department of Mathematics, University of Uppsala, Uppsala, Sweden

Albert Edrei
The Department of Mathematics, Syracuse University, Syracuse, New York 13210

Fred Gross
The Department of Mathematics, University of Maryland at Baltimore County, Md. 21228 and Naval Research Laboratory, Washington, D.C. 20375

Yong-Xing Gu
The Department of Mathematics, Nan-Chun Normal College, Sze-Chwan Province, China

Yu-Zan He
Institute of Mathematics, Academia Sinica, Beijing, China

Simon Hellerstein
The Department of Mathematics, University of Wisconsin, Madison, Wisconsin 53706

Jun-Shung Hwang
Institute of Mathematics, Academia Sinica, Taiwan

James K. Langley
The Department of Mathematics, University of Illinois, Urbana, Illinois 61801

Peter A. Lappan
The Department of Mathematics, Michigan State University, Lansing, Michigan 48823

Lennox S.O. Liverpool
The Department of Mathematics, Jos University, Jos, Nigeria

Seiki Mori
The Department of Mathematics, Yamagata University, Yamagata, Japan

Kiyoshi Niino
Faculty of Technology, Kanazawa University, Kanazawa 920, Japan

Charles F. Osgood
Naval Research Laboratory, Washington, D.C. 20375

John Rossi
The Department of Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

Ranjan Roy
The Department of Mathematics, Beloit College, Beloit, Wisconsin 53511

S.M. Shah
The Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506

Daniel F. Shea
The Department of Mathematics, University of Wisconsin, Madison, Wisconsin 53706

Li-Chien Shen
The Department of Mathematics, The California Institute of Technology, Pasadena, California 91125

Leonard M. Smiley
The Department of Mathematics, University of Alaska, Anchorage, Alaska 99506

Guo-Dong Song
The Department of Mathematics, Cornell University, Ithaca, New York 14853

Charles S. Stanton
The Department of Mathematics, University of North Carolina, Chapel Hill, North Carolina 27514

Shlomo Strelitz
The Department of Mathematics, University of Haifa, Haifa, Israel

Nobushige Toda
The Department of Mathematics, Nagoya Institute of Technology, Nagoya, Japan

Sakari Toppila
The Department of Mathematics, University of Helsinki, Helsinki 10, Finland

Chia-Chi Tung
The Department of Mathematics, Astron. and Statistics, Mankato State University, Mankato, Minnesota 56001

Jack Williamson
The Department of Mathematics, University of Hawaii, Honolulu, Hawaii 96822

Xiuzhi Xiao
The Department of Mathematics, Wuhan University, Wuhan, China

Nino Yanagihara
The Department of Mathematics, Chiba University, Chiba, Japan

Chung-Chun Yang
Naval Research Laboratory, Washington, DC 20375

Kenneth B. Yuguda
The Department of Mathematics, University of Jos, Jos, Nigeria