

# CONTEMPORARY MATHEMATICS

## Contributions to Group Theory

AMERICAN MATHEMATICAL SOCIETY

**VOLUME 33**

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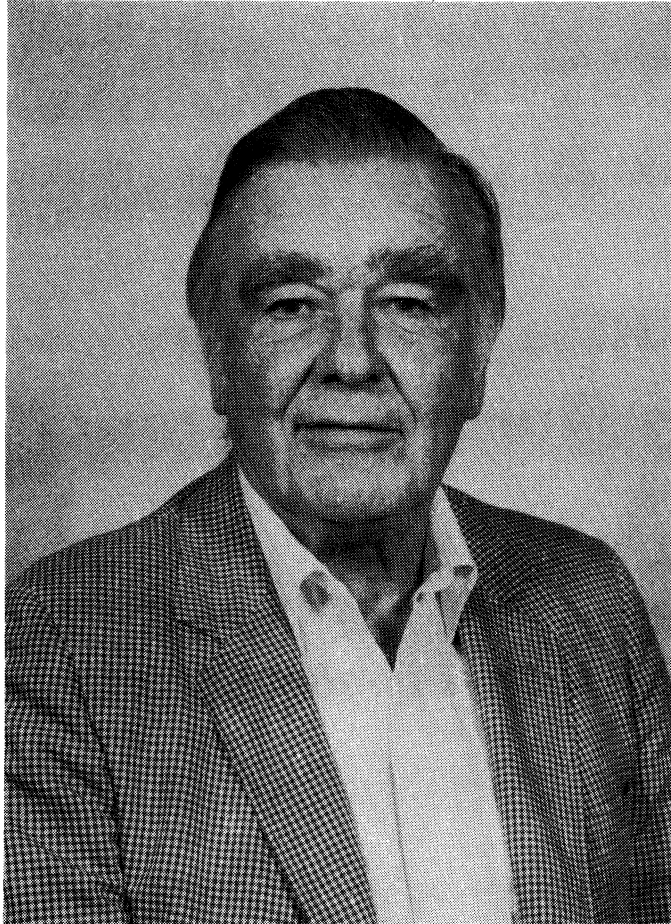
**Volume 33**

## **Contributions to Group Theory**

**Kenneth I. Appel, John G. Ratcliffe  
and Paul E. Schupp, Editors**

**AMERICAN MATHEMATICAL SOCIETY**

**Providence • Rhode Island**



Papers dedicated to  
ROGER C. LYNDON  
on the occasion  
of his sixty-fifth birthday.



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## Preface

This volume consists of five short articles on Roger Lyndon and his contributions to mathematics, and twenty-seven invited research papers. The research articles involve topics in combinatorial group theory and closely related areas. Several of the articles fall into subfields of combinatorial group theory in which much of the initial work was done by Lyndon. Most of the rest are in areas in which he has made major contributions. It is a tribute to Lyndon's mathematical breadth that papers covering such a wide array of topics are all closely related to the work he has done.

The articles on Lyndon and his work include a biographical essay by Kenneth Appel and expository articles by Saunders Mac Lane, John Ratcliffe, Jerome Keisler, and Paul Schupp. Mac Lane describes the results in Lyndon's doctoral thesis and explains how they fit into the early history of spectral sequences. Ratcliffe describes Lyndon's fundamental work in cohomology of groups in the early part of his career. Keisler discusses Lyndon's work in logic, especially his fundamental results in model theory in the mid 1950s. Schupp then describes Lyndon's work in group theory over the past twenty years.

The volume grew out of a desire by the editors to honor our teacher, Roger Lyndon, on the occasion of his sixty-fifth birthday. It has been a labor of love for us to gather together papers of solid mathematical interest dedicated to Roger Lyndon by his students, colleagues and friends, all of whom have shown great enthusiasm for this project.

We would like to thank all of the contributors and all the people who have made this collection possible. All the papers in this volume have been referred.

Kenneth I. Appel  
John G. Ratcliffe  
Paul E. Schupp

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