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AMERICAN MATHEMATICAL SOCIETY

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Volume 34

Combinatorics and Algebra

Curtis Greene, Editor

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PROCEEDINGS OF THE AMS-IMS-SIAM JOINT SUMMER RESEARCH CONFERENCE IN THE MATHEMATICAL SCIENCES ON COMBINATORICS AND ALGEBRA

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INTRODUCTION

This volume contains the Proceedings of the AMS-NSF Joint Summer Research Conference on COMBINATORICS AND ALGEBRA held at the University of Colorado, Boulder, during June 5-11, 1983. The conference organizing committee consisted of Adriano Garsia, Curtis Greene, Gian-Carlo Rota, and me.

Although combinatorial techniques have pervaded the study of algebra throughout its history, it is only in recent years that any kind of systematic attempt has been made to understand the connections between algebra and combinatorics. Both subjects have been greatly enriched as a result of this undertaking. The present Research Conference drew together specialists in both algebra and combinatorics, and provided an invaluable opportunity for them to collaborate.

The topic most discussed at the conference was representation theory, in particular the representation theory of the symmetric group and complex general linear group. The close connections with combinatorics, especially the theory of Young tableaux, was evident from the pioneering work of G. Frobenius, I. Schur, A. Young, H. Weyl, and D. E. Littlewood. In response to popular demand Phil Hanlon gave an introductory survey of this subject, whose inclusion in this volume should make many of the remaining papers more accessible to a reader with little background in representation theory.

Ten of the papers in this volume (excluding Hanlon's survey) impinge on representation theory in various ways. Some are directly concerned with the groups, Lie algebras, etc., themselves, while others deal with purely combinatorial topics which arose from representation theory and suggest the possibility of a deeper connection between the combinatorics and the algebra.

The remaining papers are concerned with a wide variety of topics. There are valuable surveys on the classical subject of hyperplane arrangements and its recently discovered connections with lattice theory and differential forms, and on the surprising connections between algebra, topology, and the counting of faces of convex polytopes and related complexes. There also appears an instructive example of the interplay between combinatorial and algebraic properties of finite lattices, and an interesting illustration of combinatorial reasoning to prove a fundamental algebraic identity.

In addition to the talks represented by papers in this volume, there were three other stimulating lectures by I. G. Macdonald, David Anick, and Ranee Gupta. A highly successful problem session was held during the conference, and a list of the problems presented appears at the end of this volume.

As chairman of the Organizing Committee for the Research Conference I wish to thank the American Mathematical Society for its support, and especially Mrs. Carole Kohanski for her unending efforts to insure that the conference was a success. I am also grateful to Curtis Greene for serving as editor of these Proceedings.

Richard P. Stanley
Cambridge, MA
July, 1984

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ADDENDUM

Several proposers have provided additional information concerning problems submitted to the problem session, including solutions obtained since the Boulder meeting. The following summarizes all information received by the editor as of press time (August 1984):

PROBLEM 1 (S. Yuzvinsky) was proposed independently by P. Frankl (Marseille 1981, Oberwolfach 1982), and is still open.

PROBLEM 4 (R. Stanley) was solved by D. Worley, and the solution appears in his Ph.D. thesis (M.I.T. 1984).

PROBLEM 5 (P. Hanlon, R. Gupta) was solved by J. Stembridge, and the solution will appear in his M.I.T. Ph.D. thesis.

PROBLEM 18 (G.C. Rota) was solved by D. Coppersmith, and the solution will appear in *Advances in Mathematics*.

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