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Analytic Functions of One Complex Variable

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Volume 48

Analytic Functions of One Complex Variable

Chung-chun Yang and
Chi-tai Chuang, Editors

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PREFACE

In the past, Chinese scholars had won brilliant accomplishments with inventions and creations in the evolution of science and technology, and particularly, made significant contributions in earlier development of mathematics.

Unfortunately, for about two decades, before and during the disruptive cultural revolution, the Chinese scientific community as a whole had been alienated from the western world and publication of works by Chinese scientists was very much discouraged.

Now, since the four modernization plans, China has reopened her doors and has rapidly taken the path to scientific and cultural interactions with the rest of the world

As mathematicians, we have planned to edit a series of mathematical monographs in various fields of mathematics: Complex Analysis, Harmonic Analysis, Functional Analysis, Differential and Integral Equations, Algebra, Topology and Differential Geometry, Number Theory, Numerical Analysis, and Applied Mathematics.

It is hoped that these collections will not only reflect the achievements and contributions of Chinese Mathematicians in the development of contemporary mathematics, but, more importantly, provide a broad view of the scope of and aid greatly those interested in keeping abreast of developments in mathematical research in modern China for the past three decades. Furthermore, we hope the publication of these volumes will further promote the scientific exchanges between West and East, and become a trigger for similar projects in other scientific disciplines.

Thus, the nature of the articles solicited are mostly expository and survey types and written by contemporary distinguished Chinese mathematicians in their respective fields. The theme presented in this volume is focused on complex analysis of one variable, which is a well developed field in China.

In presenting this first volume to the reader, we wish to thank the American Mathematical Society for their support and endorsement in such an endeavor; which also made this difficult task an enjoyable and rewarding experience.

Finally, allow us in the name of the Editors to express our heartfelt thanks and appreciation to all the contributors who have kindly prepared and sent us their articles toward the completion of this book.

Chung-chun Yang 楊重駿

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