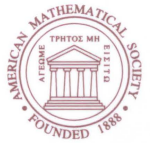
CONTEMPORARY MATHEMATICS

55, Part 1

Applications of Algebraic K-Theory to Algebraic Geometry and Number Theory

Proceedings of a Summer Research Conference
June 12–18, 1983

Spencer J. Bloch
R. Keith Dennis
Eric M. Friedlander
Michael R. Stein
Editors



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INTRODUCTION

During the week of June 12-18, 1983, a research conference on APPLICATIONS OF ALGEBRAIC K-THEORY TO ALGEBRAIC GEOMETRY AND NUMBER THEORY was held at the University of Colorado in Boulder under the auspices of the American Mathematical Society with funding from the National Science Foundation. This volume contains the Proceedings resulting from that conference.

The diversity of recent work in pure and applied algebraic K-theory is evident from the talks given at the conference as well as the papers in this volume. Although algebraic K-theory grew from topology and the ideas of Grothendieck on the Riemann-Roch theorem, it acquired a character of its own from the algebraic work initiated by Bass, Swan and Milnor, which focused in large part on matrices and matrix-related constructions. Bass and Tate realized the significance of K_2 for arithmetic. Then Quillen gave an extraordinarily flexible reformulation of the foundations of the subject which paved the way for rich and varied interactions with algebraic geometry and topology.

The matrix/vector bundle tradition of concrete computations for specific rings is represented here, for example, by the papers of Berrick-Keating, Dwyer-Friedlander, Hurrelbrink-Kolster, Rehmann, Swan-Vaserstein, Vaserstein and Weibel. It is on these foundations that general conjectures are formulated and tested.

Much of the interaction with algebraic geometry focusses now on algebraic cycles rather than vector bundles; the papers of Pedrini-Weibel and Kato exemplify this. This approach shades off into arithmetic: Kato-Saito study the arithmetic of 0-cycles on arithmetic schemes, providing a generalization of Artin reciprocity by interpreting splitting of primes in abelian coverings of schemes of finite type over Z in terms of rational equivalence of 0-cycles. The analogous class field theory for curves over local fields is discussed by Coombes.

An important new direction in K-theory and arithmetic involves generalizing the regulator map for units (read "K_1") in an

algebraic number field to the higher K-groups of varieties over number fields. A conjecture of Beilinson interprets certain transcendental numbers obtained in this way in terms of the values of L-functions. These ideas are discussed in the papers of Beilinson, Bloch, Bloch-Grayson, Ramakrishnan, and Wagoner.

The important advances in the K-theory of fields made by Merkurjev and Suslin are reported on in the papers by Dwyer and Friedlander, Merkurjev, and Wadsworth. Many other topics ranging from the connections of K-theory with Hochschild homology and cyclic homology to the applications of K-theory to sums of squares were discussed at the conference. Although all topics discussed are not represented in these Proceedings, ample material remains to occupy the interested reader.

The success of this conference is due to the hard work of many individuals and the generous support of two organizations. We would like to take this opportunity to thank them. The National Science Foundation provided financial support while the American Mathematical Society offered its organizational skills and support staff. The University of Colorado in Boulder provided the beautiful surroundings in which the conference took place. Ronnie Wells gave his help during the planning of the conference. The efforts of Carole Kohanski were indispensable.

The Editors

LIST OF TALKS

Invited Addresses

W. G. Dwyer, Conjectural calculations of the cohomology of general linear groups.

J.-L. Loday, Homology of Lie algebras of matrices and cyclic homology.

S. Bloch, Algebraic K-theory and L-functions, I.

A. Merkurjev, K_2 of fields and the Brauer group.

C. Soulé, P-adic K-theory of elliptic curves.

W. G. Dwyer, A survey of Suslin's recent work on the Lichtenbaum conjectures.

K. Kato, Algebraic K-theory and class field theory.

A. Merkurjev, A survey of Suslin's work on the torsion in K_2 of fields.

S. Bloch, Algebraic K-theory and L-functions, II.

R. Thomason, Bott stability in algebraic K-theory.

R. Charney, Excision and the K-theory of orders.

W. Van der Kallen, Relative K_2 of truncated polynomial rings.

D. Kazhdan, Analytic K-theory.

J. Wagoner, Transcendental and étale p-adic regulators.

Special Sessions

K-Theory and Algebraic Geometry

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J. Stienstra, Cartier Dieudonné theory for chow groups.

J. Murre, Application of Mercurjev-Suslin to algebraic cycles

C. Pedrini, K-theory and chow groups on singular varieties.

B. Dayton, K_0 regularity and seminormality.

S. Landsburg, K-theory and relative cycles.

Y. Nisnevich, Conjecture on rationally trivial torsions.

K. Kato, Relative 0-cycles and Lang's class-field theory.

Special Sessions

Connections with Topology

H. Gillet, K-theory of local hensel rings.

C. Weibel, Pedersen's developing of K-theory.

J. Jardine, Simplicial objects in a Grothendieck topos.

Z. Fiedorowicz, Hermitian K-theory of simplicial rings.

R. Charney, Compactifications of moduli spaces.

L. Vaserstein, On K_1-theory of topological spaces.

V. Snaith, Algebraic K-theory and topological K-theory.

D. Burghelea, Homotopical applications of hermitian algebraic K-theory.

Cohomology and K_1

B. Magurn, Injective stability for cyclic groups.

S. Geller, Subgroups of congruence level I^2.

T. Vorst, Some stability results for K_1.

C.-H. Sah, Schur multipliers for classical Lie groups.

R. Oliver, An exact sequence involving $K_2(\hat{\mathbb{Z}}_p\pi)$, $K_1(\hat{\mathbb{Z}}_p\pi)$.

D. Grayson, Semistability and reduction theory.

M. Krusemeyer, Possible computations of SK_1 for plane cubic curves.

J. Huebschmann, The topology of $F\Psi^q$ and $BGL(\mathbb{F}q)^+$.

Number Theory

A. Kuku, K-theory of group rings of finite groups over maximal orders in division algebras.

D. Goss, Arithmetic theory of algebraic curves over finite fields.

M. Kolster, On the Birch-Tate conjecture for maximal real subfields of cyclotomic fields.

J. Hurrelbrink, On the orders of $K_2(O)$ in some cyclotomic cases.

S. Rosset, A reciprocity formula for K_2-traces.

J. S. Hsia, An application of K-theory to sums of squares.

J. Queyrut, Galois modules structures.

U. Rehmann, A metaplectic theorem for certain anisotropic groups.

A. Bak, K_2-analogs of Hasse's norm theorems.

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