CONTEMPORARY MATHEMATICS

Multiparameter Bifurcation Theory

Proceedings of a Summer Research Conference held July 14–20, 1985

AMERICAN MATHEMATICAL SOCIETY

VOLUME 56

http://dx.doi.org/10.1090/conm/056

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Multiparameter Bifurcation Theory

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Volume 56

Multiparameter Bifurcation Theory

Proceedings of the AMS-IMS-SIAM
Joint Summer Research Conference
held July 14–20, 1985, with support
from the National Science Foundation

Martin Golubitsky and
John M. Guckenheimer, Editors

AMERICAN MATHEMATICAL SOCIETY
Providence • Rhode Island

The AMS-IMS-SIAM Joint Summer Research Conference in the Mathematical Sciences on Multiparameter Bifurcation Theory was held at Humboldt State University, Arcata, California on July 14–20, 1985 with support from the National Science Foundation, Grant DMS-8415201.

1980 *Mathematics Subject Classification*. Primary 58F14, 34C35, 76E30.

Library of Congress Cataloging-in-Publication Data

Multiparameter bifurcation theory.

(Contemporary Mathematics; v. 56)

Bibliography: p.

I. Bifurcation theory–Congresses. I. Golubitsky, Martin, 1945– . II. Guckenheimer, John. III. American Mathematical Society. IV. Institute of Mathematical Statistics. V. Society for Industrial and Applied Mathematics. VI. Series: Contemporary mathematics (American Mathematical Society); v. 56.

QA372.M885 1986 515.3′52 86-8106

ISBN 0-8218-5060-1, ISSN 0271-4132

Printed in the United States of America.
This volume was printed directly from author prepared copy.
The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability. ♾
10 9 8 7 6 5 4 3 2 95 94 93 92 91 90

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ABSTRACTS

PREFACE

This volume contains the proceedings of an American Mathematical Society Summer Research Conference held in Arcata, California during July, 1985. Bifurcation theory has been an area of intense interdisciplinary research and it is our hope that this volume will convey the excitement that we have found coexisting with this interaction. The meeting brought together scientists working on fluid instabilities and chemical reactor dynamics as well as mathematicians interested in multiparameter bifurcation. These two areas of applications represent ones in which experiment and mathematical analysis mutually enhance each other. The experimental and numerical results have suggested interesting and fruitful mathematical problems which in turn have made predictions to be tested by experiments. The subject is far from closed; work continues with the goal of making the mathematics, physics and chemistry fit together more completely.

Multiparameter bifurcation theory seeks to classify the transitions which may be expected to occur in dynamical systems as several parameters are varied. Such classifications are based on the mathematical notion of genericity; genericity, however, depends crucially on the particular context. Various papers in these proceedings study steady state bifurcation, Hopf bifurcation to periodic solutions, interactions between modes, dynamic bifurcations and the role of symmetries in such systems. The mathematics offers hope, at least, for a systematic classification of the possible bifurcations in these settings.

The theoretical and experimental studies of chemical reactors and fluids provide many situations where these mathematical ideas may be tested. Stirred tank chemical reactors may have several infeeds where different compounds are fed into the reactor at different rates and at different temperatures. The goal of theory is to predict the types of transitions in the (asymptotic) state of the reactor as these controls are changed. Even the analysis of the simplest mathematical model (the CSTR) has provided chemical engineers with research problems for generations.

Rutherford Aris has described the CSTR by quoting from Macbeth "Yet who would have thought the old man to have had so much blood in him". Recently, however, the new methods of bifurcation theory have aided significantly in the analysis of the CSTR.

In fluid dynamics, substantial effort has been expended on determining the routes to chaos in two laboratory systems: Taylor-Couette flow between rotating cylinders and Rayleigh-Benard convection in a fluid layer. Most of this work is devoted to studying routes to chaotic behavior occurring with the systematic variation of one experimental parameter. Recently however, a number of experiments have been performed which carefully document the states of these systems when two parameters are varied. Several of the participants of this meeting have performed these experiments and are actively trying to compare their results with the mathematical predictions of codimension two bifurcation theory. These efforts are complicated both by the presence of symmetry and by imperfections that break those symmetries.

In the Couette-Taylor experiment especially, many of the observed states enjoy symmetries which vary from state to state. This rich structure has yielded a moderately long list of problems that currently challenge the theorists. The theoretical program consists of understanding mathematically the individual transitions and symmetries that are observed, computing appropriate normal forms, determining the values of coefficients in the normal form directly from the fluid equations, and finally elucidating the dynamics associated with these normal forms. The hope is that the structure of the bifurcations found in these model equations will correspond to those observed in experiments.

For Rayleigh-Benard convection a similarly rich structure of fluid states has been observed both in experiment and in the mathematical analysis. Here however, the boundary conditions used in theoretical studies seldom correspond fully to those of experiments. For example, in the experiments described in this volume, heat flow through sidewalls represents a significant departure from the symmetry of the system with idealized boundary conditions. Finally, the Rayleigh-Benard problem serves as a wonderful test problem for other diffusive and doubly diffusive phenomena.

The mathematical study of multiparameter bifurcation theory presents a variety of theoretical and practical difficulties, many of which are discussed in these proceedings. Bifurcation problems with the exception of the simplest problems lead to specific systems of differential equations of moderate size. When these problems come from systems with symmetry, even the task of enumerating the equilibrium solutions of the normal form equations is a substantial task to which the sophisticated tools of singularity theory and group theory can be applied productively. The study of nonequilibrium behavior in these problems is even more complicated. The delicacies of the nonlinear interactions involved in higher codimension bifurcations seem at times diabolically designed to frustrate attempts to determine unfoldings by means of numerical integration. There are techniques however, which do aid in such attempts and many of these are used in this volume.

All of the contributed papers appearing in these proceedings have been refereed. Our guidelines were that the contributed papers should contain either original research or be surveys that go beyond being just synopses of other papers. Abstracts from several of the lectures have been included to serve as guides and pointers to the literature.

It remains only to thank the many people whose efforts made both the conference and these proceedings possible. Edgar Knobloch and Dan Luss actively helped plan the conference program. The referees' efforts did help improve the papers appearing in this volume. Carole Kohanski simplified the mechanics of organizing, running and participating in the conference. Most importantly, we thank the participants and the contributors.

Martin Golubitsky
Houston, Texas

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Ithaca, New York

February, 1986

Invited Lectures

F. H. Busse	Rotating fluids
Pascal Chossat	Interaction of rotating waves in the strongly counterrotating Couette-Taylor problem
Pierre Coullet	Quasiperiodic patterns
Gerhard Dangelmayr	Stationary bifurcations in non-flux boundary value problems
Thomas Erneux	Slowly varying bifurcation parameters: Hopf bifurcation; bursting oscillations
Martin Golubitsky	An introduction to singularity theory
Brian F, Gray	Explosion theory and singularity theory
Jack K. Hale	Flows on center manifolds for delay equations
Vladimir Hlavacek	Examples of bifurcations in the theory of reacting systems
Pierre Hohenberg	Modulated convection: Theory and experiment
Allan Jepson	The numerical computation of singular points
Edgar Knobloch	Doubly diffusive waves
William F. Langford	Multiparameter bifurcation and symmetry in Taylor-Couette flow
Dan Luss	Steady state multiplicity of chemically reacting systems
Irene Moroz	Development of spatial complexity in Langmuir circulations
Leonid Pismen	Selection of wave patterns through mode interaction: oscillatory Marangoni instability
M. R. E. Proctor	Convection at large and small aspect ratio
Ingo Rehberg	Experimental observation of a codimension two bifurcation in a binary fluid mixture
Mark Roberts	Degenerate Hopf bifurcation with O(2) symmetry
Jan Sanders	Nilpotent normal forms and representation theory of sl(2,R)
David H. Sattinger	Hamiltonian hierarchies on semisimple Lie algebras
Ian Stewart	Hopf bifurcation with symmetry
Randall Tagg	"Multicritical points" in flow between independently rotating cylinders

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ISBN 0-8218-5060-1