

CONTEMPORARY MATHEMATICS

Current Trends in Arithmetical Algebraic Geometry

**Proceedings of a Summer
Research Conference held
August 18–24, 1985**

AMERICAN MATHEMATICAL SOCIETY

VOLUME 67

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in Arithmetical
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**Proceedings of the AMS-IMS-SIAM
Joint Summer Research Conference
held August 18–24, 1985 with support
from the National Science Foundation**

Kenneth A. Ribet, Editor

AMERICAN MATHEMATICAL SOCIETY

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INTRODUCTION

This volume is the record of a 1985 week-long Joint Summer Research Conference on Algebraic Geometry, held in Arcata, California. The conference, organized by Michael Artin, Barry Mazur, and myself, focused on three current developments in our field:

1. The work of J-M. Fontaine and W. Messing linking p-adic étale cohomology with crystalline cohomology.
2. Diophantine conjectures motivated by hyperbolic geometry.
3. Arakelov theory and its generalization to higher dimensions.

A majority of the lectures given at the conference fell clearly into one of these categories: the lectures of Messing concerned the first topic, those of S. Kobayashi, S. Lang and P. Vojta the second, and those of H. Gillet, C. Soulé and L. Szpiro the third. Other talks at the conference, such as those of G. Anderson, R. Livné and J. Silverman, cannot be so neatly classified.

The manuscripts assembled for this volume include six speakers' contributions, plus four additional papers which do not correspond to talks at the conference:

1. A. Beilinson's article on Height Pairings, frequently cited during the conference, which has been unpublished until now.
2. An updated version of notes by P. Deligne and D. Husemöller on Drinfel'd modules which were written several years ago but never published.
3. A new manuscript of Deligne, "Le déterminant de la cohomologie," which grew out of a letter of Deligne to Quillen that was discussed by Gillet in his lectures at the conference.
4. J-P. Serre's letter to J-F. Mestre (July, 1985) concerning mod p representations of $\text{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ which arise from modular forms. The "epsilon" of this letter is the extra information needed to realize G. Frey's idea that Fermat's "Last Theorem" follows from the conjecture that every elliptic curve over \mathbb{Q} is a factor of some Jacobian $J_0(N)$. I am currently preparing a manuscript which should prove a slightly weakened version of "epsilon," thus in fact establishing the link which was hoped for by Frey. For precise conjectures which

predict that all irreducible mod p representations of $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ with odd determinant arise from mod p modular forms, see Serre's article, *Sur les représentations modulaires de degré 2 de $\text{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$* , Duke Math. J. 54, 179-230 (1987).

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