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Number Theory and Its Applications in China

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Wang Yuan, Yang Chung-chun, and Pan Chengbiao, Editors

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Dedicated to the memory of

Hua Lookeng 华罗庚

—— a superb researcher and a great teacher

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PREFACE

Number theory is one of the earliest explored modern mathematical fields in China. Undoubtedly, among all the mathematical disciplines pursued by Chinese, it is also the most accomplished one.

It was first introduced into China in the early 1920's by Yang Wu-zhi. The most important pioneers in the development of this field in China are Hua Loo-keng, Ko Chao, and Min Si-he. Particularly noted is Hua, who, from 1936-1938, visited Cambridge University of England and studied under G.H. Hardy. His works in the estimation of complete exponential sums, Warings problems, Tarry's problems, and Vinogradov's method constitute a significant contribution to the development of number theory. These accomplishments also caused him to be regarded by his contemporaries as one of the world's leading number theorists of his generation.

After the founding of the People's Republic of China, Hua was appointed as the director of the Institute of Mathematics, Academia Sinica. There, he organized two seminars: one on Goldbach's problem and the other on elementary number theory. Min Si-he organized a seminar on analytic number theory at Peking University. Ko Chao led a team doing reserach on Diophantine equations at Si Chuan University. As a result of these events, many competent young mathematicians came to the fore and the most noted accomplishments by Chinese mathematicians were focused toward the solution of the Goldbach conjecture and on sieve method. For instance, based on the work of Wang Yuan and Pan Cheng-dong, Chen Jing-run was able to prove that every sufficiently large even number can always be expressed as the sum of a prime number and an integer having at most two prime factors.

Unfortunately, the cultural revolution interrupted the research of number theory in China for more than ten years. After that the study of number theory has been resumed. Now besides Beijing and Si Chuan, the Shandong University and Chinese University of Science and Technology also have advanced programs in number theory. More specifically, in addition to the old topics, a wide variety of subjects in diophantine approximations, transcendental number theory, uniform distribution, modular forms, and algebric number theory have been taken up by middle age scholars. Chinese number theorists also have been interested in the applications of number theory.

This book contains nine survey articles and three articles of authors on recent research work. It mainly covers research topics relating: the circle method and the methods of estimating the exponential sums, Waring's problem and Goldbach's problem, weighted sieve method and mean value theorems, Riemann zeta function, arithmetic function, Dirichlet L-functions, number theoretic methods in numerical analysis, diophantine equations and diophantine inequalities in algebraic number fields, diophantine approximations and transcendental number theory, modular forms, and applications of number theory to digital signal processing and public-key systems.

Notice that in this collection we emphasize the introducing and gathering of research accomplishments of Chinese number theorists during 1949-1979, a period of time during which correspondence between Chinese and foreigners was discouraged. We also remark that here and in the text, the Chinese authors' names are spelled as they should be called in China by putting the surname before the first name.

The contributors to this volume are leading experts in China. It is hoped that the collection will not only survey the significant contributions of Chinese mathematicians, but also reflect broadly the latest developments and current state of the research of number theory in China. Thus a more concrete scientific exchange between East and West in this area will result.

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