

CONTEMPORARY MATHEMATICS

Abelian Group Theory

Proceedings of the
1987 Perth Conference
held August 9–14, 1987

AMERICAN MATHEMATICAL SOCIETY

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Helmut Ulm
1908-1975

Abelian Group Theory

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Volume 87

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**Laszlo Fuchs, Rüdiger Göbel,
and Phillip Schultz, Editors**

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Preface

The traditional biennial international conference of abelian group theorists was held August 9-14, 1987 at The University of Western Australia, in Perth. In spite of the large distance from most research centres of abelian group theory, the conference was well-attended: some 40 participants came from five continents. A welcome addition was the attendance of several ring theorists.

The papers in this volume are those presented at the conference and a few others submitted by colleagues unable to attend. They indicate the healthy state of abelian group theory; in most branches of the subject, significant advances have been made since the previous conference at Oberwolfach in 1985.

The first section is concerned with the structure of p -groups, starting with a survey by Göbel on H. Ulm's contributions to abelian group theory and related areas – this is the third report in a series dealing with the old masters of the subject (the first were on R. Baer and H. Prüfer). The paper of Benabdallah, Cutler and Mader deals with the problem of finding the extensions of one given group by another. The authors determine the isomorphism classes of extensions of a torsion-complete p -group by $\mathbf{Z}(p^\infty)$. Honda defines a new type of basis which provides a necessary condition for a valued vector space over $\mathbf{Z}(p)$ to be isometric to the socle of a reduced p -group.

Keef's paper is a further contribution to the surprising interaction between set theory and the structure of abelian p -groups. He shows that the existence of reduced C_Ω -groups of length Ω is equivalent to Kurepa's Hypothesis. Similarly, Cutler, Mader and Megibben prove that $V = L$ implies the existence of essentially indecomposable weakly ω_1 -separable but not p^{w+1} -projective separable abelian p -groups. In a similar vein, Eda shows that the existence and properties of certain radicals is a consequence of Vopěnka's principle.

The following sequence of papers is concerned with the automorphism groups and the endomorphism rings of abelian groups. Liebert extends his recent work on the isomorphism of p -groups G and G' ($p \neq 2$) which have isomorphic automorphism groups to classify all isomorphisms between $\text{Aut } G$ and $\text{Aut } G'$. In a survey of recent work on the endomorphism rings of mixed groups, May describes realization and isomorphism theorems. Schultz finds a representation of the endomorphism ring of an indecomposable finite valuated p -group. Arnold and Vinsonhaler prove a realization theorem for quasi-endomorphism rings of an important class of Butler groups: every finite-dimensional rational division algebra is the quasi-endomorphism algebra of a B_0 -group.

A sequence of papers on torsion-free groups follows. In a significant new approach to the classification of Butler groups, Arnold studies the connection between representations of a poset and various classes of abelian groups. He thereby clarifies a puzzling similarity between the structures of indecomposable finite valuated p -groups and indecomposable Butler groups. One consequence of Arnold's result is a catalogue of Butler groups corresponding to representations of finite type. In a related area, Göbel and Sengelhoff present a realization theorem for infinite domains: every infinite domain R is essentially the endomorphism ring of a free R -module with four distinguished submodules.

In a section on the structure theory of torsion-free groups, Albrecht generalizes important properties of rank 1 groups to indecomposable groups with Noetherian hereditary endomorphism rings. Mutzbauer unifies and extends several results on the invariance and realization of various properties of the type lattice of a group. Albrecht and Hill present a new solution to the problem of characterizing separable vector groups. Metelli studies the properties of bihomogeneous groups, a natural class of groups lying between completely decomposable and separable. Goeters and Reid discuss the relationship between the invariants of torsion-free groups A and B and that of the derived groups $\text{Hom}(A,B)$ and $\text{Ext}(A,B)$.

A related pair of papers on torsion-free groups with many automorphisms follows. Dugas and Hausen examine the relations among the properties E -uniserial, E -transitive, strongly homogeneous and separable over the centre of E , where E is the endomorphism ring. Dugas and Shelah disprove a long-standing conjecture that every E -transitive group is strongly homogeneous, assuming $V = L$.

Only one paper in this volume is specifically about mixed groups. Hill and Megibben find necessary and sufficient conditions for two isotype subgroups of a Warfield group G to be equivalent under an automorphism of G and apply this result to describe complete invariants for a class of countable mixed groups.

Finally a sequence of papers deals with properties of modules which generalize corresponding properties of abelian groups. Pierce surveys E -modules over a commutative ring R , that is, R -modules A for which every additive homomorphism of R into A is an R -homomorphism. Fuchs shows that the theory of divisible modules and the problems of Baer and Whitehead extend to certain modules over a commutative domain and especially over a valuation domain. Salce and Zanardo survey finitely generated modules over valuation domains. Fay studies torsion theories T on modules over a commutative ring and finds a Matlis duality between T -divisible T -torsion modules and T -torsion-free T -cotorsion modules. Sands discusses radicals of rings which depend only on their additive structure and Mohamed and Müller prove that all continuous modules have the exchange property.

With the exception of a few surveys, all the papers in this volume have been carefully refereed to meet the standards of established international journals. We wish to record our appreciation to the referees for their speedy and efficient assistance.

We also extend our thanks to The University of Western Australia and to its Mathematics Department which hosted the conference and provided a congenial atmosphere for this important gathering, and to the British Council and the Australian Mathematical Society for financial support. Phill Schultz did most of the organisational work: he has the lion's share in the success of the conference.

Several of the papers in this volume were typeset by their authors in \TeX . However the greater part of the typesetting was done by the secretarial staff of the Mathematics Department of The University of Western Australia. Special thanks are due for the valiant efforts of Joyce Barkley, Cath Hannay and Lynn Toh.

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