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Lie Algebras and Related Topics

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CONTEMPORARY MATHEMATICS

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Lie Algebras and Related Topics

Proceedings of a Research Conference
held May 22–June 1, 1988, with support from
the National Science Foundation

Georgia Benkart and
J. Marshall Osborn, Editors

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PREFACE

During the academic year 1987-1988 the University of Wisconsin, Madison hosted a Special Year of Lie Algebras. A workshop in August 1987 inaugurated the year's activities, and a conference on Lie algebras and related topics in May 1988 marked its end. This volume contains the proceedings of the concluding conference, which featured lectures on Lie algebras of prime characteristic, algebraic groups, combinatorics and representation theory, and Kac-Moody and Virasoro algebras. Many of the facets of recent research on Lie theory are reflected in the papers presented here. The diversity that gives Lie theory its richness and relevance has also made it difficult for us to organize the papers by topic. For that reason we have chosen to arrange the papers alphabetically by the author's name.

In 1984 Richard Block and Robert Wilson announced the classification of the finite dimensional restricted simple Lie algebras over an algebraically closed field of characteristic $p > 7$. Their announcement provided the impetus for us to bring together researchers working on the long-standing problem of determining the finite dimensional simple Lie algebras over an algebraically closed field of characteristic $p > 7$. That problem now appears to be much closer to a solution as a result of the work of the participants during the special year and afterwards, particularly that of Helmut Strade.

We would like to express our appreciation to the National Science Foundation for its support through grant #DMS-87-02928 which made the workshop, conference, and other events of the special year possible. Funds from the grant also enabled short-term visitors to come to Madison to give special seminars and colloquia and to participate in informal discussions. They joined a group of long-term visitors who spent most of the year in Madison. In addition, we thank our colleagues in the Mathematics Department of the University of Wisconsin for committing the department's resources to visitors for the special year. We are grateful too to the referees who helped in the preparation of the volume and to Diane Reppert for her excellent work in typing many of the manuscripts.

Georgia Benkart and J. Marshall Osborn

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ABSTRACTS OF TALKS

ISOTROPIC LIE ALGEBRAS OF TYPE D_4

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Suppose k is a field of characteristic 0. In recent work G.B. Seligman has shown how to construct all isotropic central simple Lie algebras of type D_4. In this talk, we describe a different construction of the isotropic D_4's. Given a 4-dimensional commutative associative separable algebra $B \backslash k$ and a scalar $\mu \neq 0 \in k$, we construct (using earlier work of J. Faulkner and the speaker) an 8-dimensional algebra with involution $(B \oplus vB, -)$ from B and μ, called a quartic Cayley algebra and then apply the Kantor Lie algebra construction to obtain an isotropic Lie algebra $K(B, \mu)$ of type D_4. We give a structural characterization of quartic Cayley algebras and use it to show that any isotropic D_4 is obtained from our construction or is isomorphic to the Lie algebra of a quadratic form. Next, using work of H.P. Allen, we give necessary and sufficient conditions for two algebras $K(B, \mu)$ and $K(B', \mu')$ to be isomorphic. We also show that our construction yields a class of D_4's that are not obtained from the classical constructions studied by N. Jacobson and by H.P. Allen.

THE ROGERS-RAMANUJAN IDENTITIES: BACKGROUND AND MOTIVATION

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Among Ramanujan's most famous discoveries are the Rogers-Ramanujan identities. Indeed Hardy says of them, "It would be difficult to find more beautiful formulae than the Rogers-Ramanujan identities". We discuss some of the history of these results as well as current research. We discuss in detail Rodney Baxter's independent discovery and proof of these results for application in statistical mechanics. By following Baxter's path we obtain reasonable motivation for a variation of the proof given by Rogers and Ramanujan.

SPECIAL FUNCTIONS IN AN ALGEBRAIC SETTING

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Macdonald's identities are multivariable extensions of Jacobi's triple product for the theta function. In one variable, Ramanujan and Bailey found extensions of the triple product identity. Just as the theta function is a discrete version of the normal integral, Ramanujan's sum is a discrete version of a beta integral, and Bailey's sum can be thought of as a discrete version of a much more complicated beta integral. One way of trying to extend Macdonald's identities is to add the extra freedom that exists when going from normal to beta integrals. S. Milne did this for A_n, and R. Gustafson has recently done this for $\widehat{B}_n$ and C_n. These identities extend Bailey's very well poised sum and are sufficiently rich to contain the Macdonald identities for all the infinite families, including BC_n and both versions of B_n and C_n. Gustafson has found the corresponding identity for G_2, but it does not contain the Macdonald identity for $\widehat{G}_2$, so further work needs to be done.

In a different direction, between the Macdonald identities for G_2 and $\widehat{G}_2$, where infinite products are on base q for G_2 and on base q for the short roots of $\widehat{G}_2$ but base q^3 for the long roots of $\widehat{G}_2$, there is the example with the products for the short roots on base q and the long roots on base q^2. The corresponding Laurent series has two nonvanishing orbits in the fundamental region. It seems likely this is general. For example, it is probably true that if one separates the roots somehow (say by lengths) and uses different bases for roots in different groups, the only cases where only one orbit under the Weyl group survives is when one has a classical affine root system. The work on $G_{2,2}$ is joint with Dennis Stanton, and is still in progress.

SOME REMARKS ON GENERALIZED INTERSECTION MATRIX ALGEBRAS AND A CONJECTURE OF P. SLODOWY

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If L is a G.I.M. algebra attached to the indecomposable matrix A, then P. Slodowy has shown that there is a homomorphism from L onto a subalgebra S of a Kac-Moody Lie algebra. Here S is the fixed point set of an involution of the Kac-Moody algebra. He showed this map is an isomorphism when the matrix A is orientable. We indicate that this is also the case when the matrix A is non-orientable, and in fact present a general result on generators and relations for fixed point sets of certain kinds of involutions of Kac-Moody algebras. This is a version of the Gabber-Kac Theorem for these algebras.

We also get that S has a filtration such that the associated graded algebra is isomorphic to the positive part of the Kac-Moody algebra, and using previous results of Benkart and Moody, get that these algebras are centrally closed. Finally, we present some examples showing some of these fixed point algebras - coming from Kac-Moody algebras of infinite type - have unique simple finite-dimensional factors.

A CHARACTERIZATION OF LIE ELEMENTS AT PRIME CHARACTERISTIC

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Recall that the free Lie algebra on a set X can be constructed as the Lie subalgebra $L(V)$ of $(TV)^-$ generated by V, where TV is the tensor algebra of a vector space V with basis X; the elements of $L(V)$ are called Lie elements. A classical theorem of Friedrichs, valid only at characteristic 0, states that $x \in TV$ is a Lie element if and only if x is primitive, i.e., $\Delta x = x \otimes 1 + 1 \otimes x$ where Δ is the comultiplication in the Hopf algebra TV. The graded dual $(TV)^*$ to TV (for simplicity assuming V is finite dimensional) is the shuffle algebra ShV^*, with multiplication given by shuffle product. Equivalent to Friedrichs' theorem is the dual version of Ree: at characteristic 0, x is a Lie element if and only if $x \in ((Ker\varepsilon^*) + (A^+)^2)^\perp$, i.e., x is annihilated by 1_A and the square of the augmentation ideal A^+ of A, where A denotes $(TV)^*$ $(= ShV^*)$. This result is generalized to arbitrary characteristic using the divided power structure (coming from the divided powers $\gamma_n b = b^n/n!$ over Z) on the shuffle algebra. Let $\Gamma^2 A$ denote the divided power square of A^+, spanned by all products ab and divided powers $\gamma_n a$ $(n > 1;\ a,\ b \in A^+)$. Call x $(\in TV)$ Γ-primitive if $x \in ((Ker\varepsilon^*) + \Gamma^2 A)^\perp$.

Theorem. At arbitrary characteristic, x is a Lie element if and only if x is Γ-primitive.

STRUCTURE OF CERTAIN SIMPLE LIE ALGEBRAS OF CHARACTERISTIC 3

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Over a field k of characteristic 3 there exist simple Lie algebras that are not analogues of known simple Lie algebras of other characteristics. The structure of some of these algebras is discussed.

The algebras $L(\varepsilon)$, $\varepsilon \in k \setminus \{0\}$, of Kostrykin's parametric family of ten-dimensional simple Lie algebras, are defined as subalgebras of a contact algebra. $L(-1)$ is classical. Some distinctive properties of $L(1)$ are pointed out.

Using a construction due to Faulkner, Freudenthal triple systems unique to characteristic 3 are used to construct simple Lie algebras. Among these algebras is one of dimension 29. It contains a subalgebra S isomorphic to $L(1)$ and an S-module whose existence shows that $L(1)$ is isomorphic to a subalgebra of an algebra of type C_4.

Finally, an 18-dimensional simple Lie algebra discovered by M. Frank is discussed. It is shown that it contains a subalgebra isomorphic to $L(1)$, is a subalgebra of a contact algebra, and is the only restricted algebra in a certain class of simple algebras of dimension $2 \cdot 3^n$ for $n \geq 2$.

SOME PROBLEMS IN REPRESENTATION THEORY

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In this survey talk, representation theory was described as the search for realizations of mathematical objects such as Lie algebras, Lie groups, algebraic groups, finite groups etc. in terms of linear algebra. The basic problem of classification of finite dimensional irreducible representations of semisimple Lie algebras over the field of complex numbers was discussed. H. Weyl's results that these representations are determined by their highest weights, and his formula for their characters were examined, along with some remarks on how these ideas have been influential in more recent work. In particular, the classification using a version of highest weights of the irreducible representations of Chevalley groups and Lie algebras associated with them over algebraically closed fields of characteristic $p > 0$ was sketched, along with similar results on the classification of irreducible modular representations of finite Chevalley groups. In conclusion, Alperin's conjecture concerning a theory of weights which would classify irreducible modular representations of arbitrary finite groups, and Lusztig's conjecture on the irreducible Brauer characters of finite Chevalley groups, were stated.

THE GELFAND-GRAEV REPRESENTATIONS OF FINITE CHEVALLEY GROUPS - BESSEL FUNCTIONS OVER FINITE FIELDS

Charles Curtis

The Gelfand-Graev representation Γ of a finite Chevalley group G is an induced representation $\Gamma = ind_U^G \psi$, where ψ is a linear character in general position of the maximal unipotent subgroup U. The Hecke algebra H_Γ associated with Γ is known to be commutative. Its irreducible representations correspond to the irreducible components of Γ. In this talk, it was conjectured that each irreducible representation of H_Γ factors through the group algebra of some maximal torus of G. This was proved by Gelfand and Graev for the groups $SL_2(F_q)$, and by Chang for the groups $GL_3(F_q)$, with q odd in both cases. In the talk, a proof of the conjecture for the groups $SL_2(F_q)$ was sketched, using the Davenport-Hasse theorem on Gauss sums to establish a crucial identity for the case of nonsplit tori. As Gelfand and Graev had observed, the functions describing the representations of H_Γ in this case are analogous to the contour integral formulas for Bessel functions.

CALCULUS OF SOME FUNCTIONS ON THE ROOT LATTICE

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We discuss the interplay of various coordinatizations of the root lattice: the root, weight, capacity, and extraneous coordinates. Various combinatorial structures are assigned to each element of the root lattice as well: a weight space, i-blocks, blocks, and a zero block. A retraction, the bracketing function, from the positive cone of the root lattice onto its Kostant cone comes into play.

Using these coordinates and structures, in a completely elementary way, we obtain deep results about Kostant's partition function, both reductive in nature using the bracketing function, and inductive results using a duality result on i-blocks, as well as a zero block formula.

Applications to the representation theory of semisimple Lie algebras are noted.

KAZHDAN LUSZTIG POLYNOMIALS AND RELATED TOPICS

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This talk is divided into three parts.

I. *Survey of Kazhdan-Lusztig theory.* Here we discuss the impact of Kazhdan-Lusztig polynomials on some interesting problems in Lie theory. These include: (i) determination of multiplicities in Jordan-Hölder series of Verma modules, (ii) geometry of Schubert varieties in generalized flag manifolds, (iii) primitive ideal spectrum of enveloping algebras. Results prior to K-L papers enable us to put the K-L papers in proper perspective. In order to define the so-called Kazhdan-Lusztig polynomials (K-L polynomials), one has to consider the structure of Hecke algebras and a special involution on them; this is discussed in brief.

II. *Parabolic theme.* Here we briefly discuss the notion of a Hecke module corresponding to parabolic subsystems (of Coxeter groups, algebraic groups, Lie algebras, etc.). One then gets a relative version of K-L polynomials. These are very different in some cases and should be interesting in their own right. A general idea for this theme is to enable one to get information on original polynomials.

III. *A conjecture.* Here we look closely into the structure of Hecke algebras and formulate a conjecture for a closed formula for K-L polynomials. We also discuss the evidence in support of this conjecture.

BANACH STRUCTURES ON LOOP GROUPS

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The purpose of this talk is twofold. First we want to construct Banach manifolds X of Grassmannian type such that $AutX$ is a Banach Lie group of type A_∞, C_∞, or D_∞ . The construction of X generalizes a finite dimensional procedure (see e.g. Loos' Irvine Lecture Notes) and works for Banach Jordan pairs for which the quasi-invertible elements are dense. The elements in the corresponding group $AutX$ have the usual "fourfold decomposition property" and $LieAutX$ is realized by polynomial maps of degree ≤ 2.

Inside X we consider $X^{(n)}$ which is defined as usual (see e.g. Segal-Wilson, IHES). Moreover, we define $(AutX)^{(n)}$ as the stabilizer of $X^{(n)}$. We show that $(AutX)^{(n)}$ is essentially isomorphic to the semidirect product of $s\ell(n, A)$, where A is a naturally defined Banach algebra of functions on the unit circle S^1, and $s\ell(2, \mathbb{R})$ where its action on $X^{(n)}$ is induced from its natural action on S^1 by diffeomorphisms.

Finally, in the second part we consider $s\ell(2, A)$, where $A \equiv$ Fourier transform of $L^1(\mathbb{R})$ and show how to derive (via a "generalized Riemann-Hilbert problem") the basic equation for all AKNS-systems. It is mentioned that by this procedure one obtains essentially all L^1-initial conditions. (Most of the work on which this talk is based is jointly with E. Neher and/or J. Szmigielski.)

DERIVATIONS AND ONE-DIMENSIONAL ABELIAN EXTENSIONS OF KAC-MOODY ALGEBRAS

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In this talk we introduce new methods and results pertaining to the cohomology theory of associative algebras with involution. Since universal enveloping algebras of contragredient algebras belong to this category, we obtain in particular general statements regarding cohomology groups of Kac-Moody algebras.

Let $\mathcal{G}$ be a not necessarily symmetrizable Kac-Moody algebra with Cartan subalgebra $\mathcal{H}$ and simple root vectors $e_1, ..., e_n, f_1, ..., f_n$. The involution of $U(\mathcal{G})$ which leaves $\mathcal{H}$ invariant and exchanges the e_i and f_i will be denoted by ω. Using the Shapovalov bilinear form in conjunction with the general results alluded to in the above, we obtain

Theorem. Let λ, $\mu \in \mathcal{H}^*$ and suppose that M is an object of the B-G-G category $\mathcal{O}$.

(1) $Ext^n_{U(\mathcal{G})}(M, L(\lambda)) \cong (Tor_n^{U(\mathcal{G})}(L(\lambda), M))^*$ $\forall n \geq 0$, where $L(\lambda)$ has the structure of a right $U(\mathcal{G})$-module by setting $m \cdot u := \omega(u) \cdot m$.

(2) $Ext^n_{U(\mathcal{G})}(L(\lambda), L(\mu)) \cong Ext^n_{U(\mathcal{G})}(L(\mu), L(\lambda))$ $\forall n \geq 0$.

Let $\mathcal{G}'$ denote the derived algebra of $\mathcal{G}$ and suppose that $\mathcal{G}$ is associated to an $(n \times n)$-matrix of rank ℓ. For any homomorphism $\lambda : \mathcal{G} \longrightarrow F$ of Lie algebras let F_λ denote the corresponding one-dimensional $\mathcal{G}$-module. The following theorem generalizes several well-known classical results concerning derivations and central extensions of finite dimensional semisimple Lie algebras:

Theorem.

(1) $H^1(\mathcal{G}, \mathcal{G})$ has dimension $(n - \ell)^2$.

(2) $H^2(\mathcal{G}', F) = (0)$.

(3) $H^2(\mathcal{G}, F_\lambda) = \begin{cases} (0) & \text{for } \lambda \neq 0 \\ \Lambda^2(\mathcal{G} \setminus \mathcal{G}')^* & \text{otherwise.} \end{cases}$

THE EXCEPTIONAL AFFINE ALGEBRA $E_8^{(1)}$ AND TRIALITY

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A striking feature of the representation theory of affine algebras is the existence of a boson-fermion correspondence for some representations of some algebras. It means that one has two independent constructions, one using Heisenberg algebras and vertex operators, the other using Clifford algebras, and an isomorphism between the two pictures. For example, one has such a situation for the four representations of $D_\ell^{(1)}$ corresponding to the endpoints of the Dynkin diagram. In this work, which is joint with Igor B. Frenkel and John F. X. Reis, we have provided a boson-fermion correspondence for $E_8^{(1)}$ involving four versions of the basic representation, one homogeneously graded, three with "$k - p$" gradings. Their direct sum is naturally described in both pictures in terms of $D_4^{(1)}$ representations, and the principle of triality for $D_4^{(1)}$ plays an important role. We strongly contrast the aspects; vertex vs. spinor, finite dimensional vs. affine, and D_4 vs. E_8. The work has applications to conformal field theory.

THE CLASSIFICATION OF LIE ALGEBRA MODULES WITH FINITE DIMENSIONAL WEIGHT SPACES

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Let $\mathcal{G}$ be a finite dimensional, complex reductive Lie algebra, and let $\mathcal{U}(\mathcal{G})$ be its enveloping algebra. If $\mathcal{H}$ is a Cartan subalgebra of $\mathcal{G}$, then let $\mathcal{M}(\mathcal{G},\mathcal{H})$ denote the category of all finitely generated $\mathcal{U}(\mathcal{G})$-modules with finite dimensional $\mathcal{H}$-weight spaces. In this abstract we give a brief sketch of a classification of irreducible modules in $\mathcal{M}(\mathcal{G},\mathcal{H})$. Details will appear in [Fe1] and [Fe2]. In [Fe1] the problem of classifying irreducible modules in $\mathcal{M}(\mathcal{G},\mathcal{H})$ is reduced to the classification of "torsion free" irreducible modules of simple Lie algebras. A module $M \in \mathcal{M}(\mathcal{G},\mathcal{H})$ is said to be a torsion free module, if for every $x \in \mathcal{G}\setminus\mathcal{H}$ and $m \in M \setminus (0)$, $dim_{\mathbb{C}}\mathbb{C}[x] \cdot m = \infty$. We show that a simple Lie algebra admits a torsion free module if and only if the algebra is of type A or C. In what follows $\mathcal{G}$ will denote a simple Lie algebra of type A or C, and n will denote the rank of $\mathcal{G}$. Next we classify all primitive ideals $AnnM$ where M is an irreducible torsion free module. This could be viewed as an approximation to the classification of irreducible torsion free modules. We now use the set, $wt\ M$, of weights of M to define a representation $\rho_M : \pi_1(\mathcal{H}^* \setminus D) \longrightarrow GL(n,\mathbb{C})$ of a certain hyperplane complement $\mathcal{H}^* \setminus D$ in $\mathcal{H}^*$. It turns out that torsion free irreducible modules can be classified by the data $(AnnM, \rho_M)$. The inverse of the map $M \longrightarrow (AnnM, \rho_M)$ is described by using an explicit geometric construction. To show that the construction does in fact yield M, first we reduce to the case where $\mathcal{U}(\mathcal{G})/AnnM$ is a ring of differential operators. Here we use a translation principle and the classification of the annihilators of irreducible torsion free modules. Then in the special case where $\mathcal{U}(\mathcal{G})/AnnM$ is a ring of differential operators, we observe that the map $M \longrightarrow \rho_M$ is bijective because it amounts essentially (i.e. after passing to an associated D-module) to a Riemann-Hilbert correspondence.

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VIRASORO ALGEBRA AND COSET CONSTRUCTIONS

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A review was given of the *coset construction* of representations of the Virasoro algebra $\widehat{v}$. Unitary irreducible highest weight representations of $\widehat{v}$ are labelled by $(c,h) \in \mathbb{R}^2$, with *either* $c \geq 1, h \geq 0$ (continuum) *or* $c = 1 - \frac{6}{(m+1)(m+2)}$, $m = 0, 1, 2, \ldots$ and h taking one of $\frac{1}{2}(m+1)(m+2)$ values (discrete series). The *Sugawara construction* of the Virasoro algebra starts with an irreducible unitary highest weight representation of an affine algebra $\widehat{\mathcal{G}}$ defined by $[T^a_m, T^b_n] = if^{abc}T^c_{m+n} + km\delta^{ab}\delta_{m,-n}$ and sets $L^{\mathcal{G}}_n = \frac{1}{\beta}(\sum_m {}^{\times}_{\times} T^a_m T^a_{n-m} {}^{\times}_{\times})$ where $\beta = 2k + Q^{\mathcal{G}}, Q^{\mathcal{G}}$ being the quadratic Casimir operator of $\mathcal{G}$ in the adjoint representation and ${}^{\times}_{\times}\ {}^{\times}_{\times}$ denotes the normal ordering. This has $c = c^{\mathcal{G}} = 2k\, dim\mathcal{G}/(2k + Q^{\mathcal{G}})$ and $rank\, \mathcal{G} \leq c^{\mathcal{G}} \leq dim\, \mathcal{G}$. The *coset − construction* proceeds by considering a pair $\mathcal{G} \supset \mathcal{H}$ and consequently $\widehat{\mathcal{G}} \supset \widehat{\mathcal{H}}$, setting $K_n = L^{\mathcal{G}}_n - L^{\mathcal{H}}_n$ so defining a Virasoro representation with $c = c_K = c^{\mathcal{G}} - c^{\mathcal{H}}$. Taking $\mathcal{G} = \widehat{su}(2)_m \times \widehat{su}(2)_1$, and $\mathcal{H} = \widehat{su}(2)_{m+1}$, the diagonal subalgebra, the suffixes denoting levels, provides all the discrete series representations.

The construction enables information about Virasoro representations to be deduced from those of the affine algebras, in particular the modular properties of characters. If $\mathcal{G} \supset \mathcal{H}$ we can consider the decomposition of representations of $\widehat{\mathcal{G}}$ at a particular level with respect to those of the direct product of $\widehat{\mathcal{H}}$ (at the reduced level) and the Virasoro algebra $v_K \equiv \{K_n, c_K\}$. This decomposition is finite if and only if $c_K < 1$ and the decomposition of $\widehat{\mathcal{G}}$ with respect to $\widehat{\mathcal{H}}$ is finite if and only if $c_K = 0$. In the latter case we can use sesquilinear modular invariant combinations of characters of $\widehat{\mathcal{G}}$ to construct sesquilinear combinations for $\widehat{\mathcal{H}}$ and, in the former, we can use such combinations for $\widehat{\mathcal{G}}$ and $\widehat{\mathcal{H}}$ to construct them for $\widehat{v}$.

Extensions of those results obtain for the super-Virasoro algebras and for more general extensions of $\widehat{v}$ which are not Lie algebras but are defined by operator product expansions.

ON KAC'S "RECOGNITION THEOREM"

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As part of a program for classifying the simple Lie algebras of prime characteristic which he outlined in the early seventies, Victor Kac proved the theorem which Richard Block and Robert Wilson refer to as Kac's "Recognition Theorem" for graded Lie algebras, and which they made use of in their recent classification of the restricted simple finite dimensional Lie algebras over algebraically closed fields of prime characteristic. It is expected that the approach used in their proof can be generalized to the non-restricted case. One potentially useful ingredient in such a generalization would be a "Recognition Theorem" without the hypothesis that the adjoint representation of the null component G_0 on the minus-one component G_{-1} of the graded algebra G be restricted. Georgia Benkart and I were able to show that Kac's theorem remains true without this hypothesis. One notion used in the proof of our Main Theorem is that of the character of a representation of a restricted Lie algebra. The character χ of the representation of G_0 on G_{-1} is zero if the representation is restricted, so our main objective is to prove that χ is zero. Our method of showing that χ disappears on a particular G_{-t} involves the construction of certain depth-one quotient Lie algebras, which we denote $B(t)$. If the one-component of $B(t)$ is not zero, we can show that the representation of the zero component on the minus-one component of $B(t)$ is restricted. That implies that the representation of G_0 on G_{-t} is restricted. We are able to show that the one-component of $B(t)$ is not zero when t is equal to $q - i$, where q is the depth of G and $i = 0, 1, 2$, or 3. In this way, we are able to prove the Main Theorem for all primes greater than 5.

VERMA BASES FOR HIGHEST WEIGHT MODULES

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Let $\mathcal{G}$ be a finite-dimensional simple Lie algebra over $\mathbb{C}$ of rank ℓ, let $\mathcal{H}$ be a Cartan subalgebra, and let $e_1, \ldots, e_\ell, f_1, \ldots, f_\ell$ be Chevalley generators. Let $\alpha_1, \ldots, \alpha_\ell$ denote the simple roots, $r_i = r_{\alpha_i}$ the reflection in α_i, and $\mathcal{W}$ the Weyl group of $\mathcal{G}$.

For $V = V(\lambda)$ a finite-dimensional irreducible $\mathcal{G}$-module of highest weight $\lambda \in \mathcal{H}^*$, D-N. Verma has proposed the following method for obtaining a basis for V. Let v^+ be a highest weight vector for V and $r_{i_n} \cdots r_{i_2} r_{i_1}$ a reduced expression for w_0, the unique element of maximum length in $\mathcal{W}$. Verma's proposal consists primarily of an algorithm for finding certain functions $U_1, \ldots, U_n$ so that the set of all elements of the form

$$f_{i_n}^{a_n} \cdots f_{i_2}^{a_2} f_{i_1}^{a_1} \cdot v^+$$

such that

$$\begin{aligned} &0 \le a_1 \le U_1 \\ &0 \le a_2 \le U_2(a_1) \\ &\quad\vdots \\ &0 \le a_n \le U_n(a_1, \ldots, a_{n-1}) \end{aligned}$$

should be a basis for V.

This algorithm does not work for all reduced expressions for w_0. To date, "good" reduced expressions have been found for the classical algebras and for G_2; no one has had any luck yet with E_6, E_7, E_8 or F_4. Furthermore, even when the algorithm does work, it is by no means obvious, that the resulting elements form a basis. That a basis is obtained has only been shown for the algebras A_ℓ, B_2 and G_2, although some progress has been made on B_ℓ ($\ell \geq 3$), C_ℓ and D_ℓ. Finally, most of these results and ideas can be extended to Verma modules and to arbitrary Kac-Moody algebras.

ON RATIONALITY PROPERTIES OF INVOLUTIONS OF REDUCTIVE GROUPS

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Let k be a field of characteristic not two and $\mathcal{G}$ a connected linear reductive k-group. Write $\mathcal{G}_k$ for the set of k-rational points of $\mathcal{G}$. By a k-involution Θ of $\mathcal{G}$, we mean a k-automorphism Θ of $\mathcal{G}$ of order two. Let $\mathcal{H} = \mathcal{G}_\Theta$ be the fixed point group of Θ. For $k = \mathbb{R}$ or an algebraically closed field, such involutions have been extensively studied emerging from different interests. Especially the interactions with the representation theory of reductive groups have been most rewarding. In this talk we present a survey on rationality problems of general k-involutions; this with an emphasis on a characterization of the double coset space $\mathcal{P}_k \setminus \mathcal{G}_k / \mathcal{H}_k$ where $\mathcal{P}$ is a minimal parabolic k-subgroup of $\mathcal{G}$. The geometry of these orbits is of importance for representation theory. We also discuss the orbit closures and dimension formulas. We also generalize the notion of Cartan involution for $k = \mathbb{R}$ to a more general setting. This leads among others to a more precise description of the double cosets $\mathcal{P}_k \setminus \mathcal{G}_k / \mathcal{H}_k$.

MODULAR INVARIANT REPRESENTATIONS

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In this talk I discuss the problem of classification of modular invariant representations of affine like and Virasoro like Lie algebras and superalgebras. The key result is a character formula for a large class of highest weight representations of a Kac-Moody algebra and superalgebra, generalizing the Weyl-Kac character formula. In the case of affine algebras, this class includes modular invariant representations of arbitrary rational level m such that $(m+g) \geq g/u$, where $u > 0$ is the denominator of m and g is the dual Coxeter number. In the case of $A_1^{(1)}$ this gives a complete classification of modular invariant representations. I discuss also in detail the modular invariant representations of the Virasoro algebra, their connection to modular invariant representations of $A_1^{(1)}$ and to the Rogers-Ramanujan identities.

REFERENCE

V.Kac and M. Wakimoto, Modular invariant representations of infinite-dimensional Lie algebras and superalgebras, Proc. Natl. Acad. Sci. USA (1988).

CONSTRUCTION OF A KAC-MOODY GROUP AND APPLICATIONS

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Let $\mathcal{G}$ be a Kac-Moody Lie algebra over a field $\mathbf{F}$ of arbitrary characteristic. Let $L(\lambda), L(\mu), L(\nu)$ be three integrable modules with highest weights λ, μ, ν. Let v, $w \in W$, and $S_{w,\lambda}$ be the corresponding Schubert variety in $\mathbf{P}L(\lambda)$.

Theorem 1. The variety $S_{w,\lambda}$ is normal, and projectively normal in $S_{w,\lambda}$.

Theorem 2. (Parthasaraty-Varadarajan-Ranga Rao Conjecture) Suppose that $\lambda + v\mu = w\nu$. Then $L(\nu)$ appears as a natural subquotient of $L(\lambda) \otimes L(\mu)$ (up to sign).

Theorem 3. The natural map $L(\lambda + \mu) \longrightarrow L(\lambda) \otimes L(\mu)$ is injective.

The three theorems result from a common technical theorem, whose proof requires three main ingredients:

1. Construction of a Kac-Moody ind-scheme group.
2. Topology of the desingularisation of a generalized Schubert variety.
3. Action of the Frobenius map on some cohomological groups (following the techniques of Metha-Ramanan-Ramanathan).

Remark. The following results were proved previously:

Theorem 1: for $\mathcal{G}$ symmetrizable and $\mathbf{F}$ of characteristic 0 (Kumar and Mathieu)

Theorem 2: for $\mathcal{G} = s\ell(n)$ (Polo)

Theorem 3: for $\mathcal{G}$ symmetrizable and $\mathbf{F}$ of characteristic 0 (Deodhar-Gabber-Kac).

Remark. Independently Kumar proved Theorem 2 for $\mathcal{G}$ semisimple and $\mathbf{F}$ of characteristic 0.

THE PRIMITIVES OF THE CONTINUOUS LINEAR DUAL OF A HOPF ALGEBRA AS THE DUAL LIE ALGEBRA OF A LIE COALGEBRA

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Definition. If H is a Hopf algebra (or bialgebra), let $C.I.(H)$ denote the set of all cofinite 2-sided ideals of H, and let $H^\circ = \{f \in H^* \mid \ ker f \text{ contains } I \text{ for some } I \in C.I.(H)\}$. H^o is a Hopf algebra, called the *continuous linear dual of H* (or, more briefly, the dual Hopf algebra of H).

Notation. If H is a Hopf algebra (or bialgebra) let $P(H)$ denote the Lie algebra of primitives of H, let $Q(H)$ denote the Lie coalgebra of indecomposables of H, and let $[Q(H)]^*$ denote the dual Lie algebra of $Q(H)$.

Theorem. For any Hopf algebra (or bialgebra) H, $P(H^\circ) \cong [Q(H)]^*$ as Lie algebras.

We discuss the following example of a nonzero Lie coalgebra M for which $Loc\, M = 0$.

Let K be a field of characteristic 0, let $W_1 = Der_K(K[x])$, and let $M = (W_1)^\circ$ be the dual Lie coalgebra of W_1. By definition $(W_1)^\circ$ is the largest subspace of $(W_1)^*$ carrying a Lie coalgebra structure induced by the restriction to $(W_1)^\circ$ of ϕ^* where $\phi = [\,,\,] : W_1 \otimes W_1 \longrightarrow W_1$ is the bracket of W_1. Then M is an example of a nonzero Lie coalgebra in which no element of M other than 0 lies in a finite dimensional sub Lie coalgebra. Thus $Loc(M) = 0$ where $Loc(M)$ denotes the sum of all the finite dimensional sub Lie coalgebras of M.

ON IRREDUCIBLE REPRESENTATIONS OF SOME AFFINE LIE ALGEBRAS

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The existence of explicit realizations of some nontrivial integrable highest weight modules is one of the most interesting features of the representation theory of affine Lie algebras. In recent years it has been established that the existence of such constructions gives rise to a number of important connections of affine Lie algebras with different branches of mathematics and physics.

In this talk we give a vertex operator realization of all level two integrable highest weight modules of an affine special linear Lie algebra of odd rank. In order to accomplish this we use the fact that the principally specialized characters of these modules differ from the product sides of the generalized Rogers-Ramanujan identities due to Gordon, Andrews, and Bressoud by a simple factor.

ON INFINITE ROOT SYSTEMS

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Definition. A set of root data over a field $\mathbf{K}$ of characteristic 0 is a 6-tuple

$$\mathcal{D} = (A, \Pi, \Pi^\vee, V, V^\vee, < \cdot, \cdot >)$$

satisfying

RD1 $A = (A_{ij})_{i,j \in \underline{J}}$ is a generalized Cartan matrix.

RD2 $V, V^\vee$ are vector spaces over $\mathbf{K}$ and $< \cdot, \cdot >: V \times V^\vee \longrightarrow \mathbf{K}$ is a nondegenerate pairing.

RD3 $\Pi = \{\alpha_j\}_{j \in \underline{J}} \subset V$, $\Pi^\vee = \{\alpha_j^\vee\}_{j \in \underline{J}} \subset V^\vee$ and $< \alpha_i, \alpha_j^\vee > = A_{ij}$.

RD4 If $Q = \sum \mathbf{Z}\alpha_i$, and $Q^\vee = \sum \mathbf{Z}\alpha_i^\vee$, then Q and $Q^\vee$ are free abelian groups and have bases $\{\gamma_i\}_{i \in \underline{I}}$ (resp. $\{\gamma_i^\vee\}_{i \in \underline{I}}$) consisting of $\mathbf{K}$-linearly independent elements such that $\Pi \subset \sum \mathbf{R}_{\geq 0}\gamma_i$, $\Pi^\vee \subset \sum \mathbf{R}_{\geq 0}\gamma_i^\vee$.

Defining the Weyl group as usual ($W \cong W^\vee$) and setting $\Sigma = W\Pi$, $\Sigma^\vee = W\Pi^\vee$, we call Σ the root system of $\mathcal{D}$ and $\mathcal{D}$ a set of root data for Σ. The authors prove that these axioms lead to a consistent theory of root systems (unlike the usual notion of realizations) in the sense that if $dim V < \infty$ then

(1) If $\Phi \subset \Sigma$ and $B := < \alpha, \beta^\vee >_{\alpha,\beta \in \Phi}$ then $\mathcal{D}' = (B, \Phi, \Phi^\vee, V, V^\vee, < \cdot, \cdot >)$ is root data for Σ if and only if $\Phi = \pm w\Pi$ for some $w \in W$.

(2) If $\emptyset \neq \Omega \subset \Sigma$ and $\alpha, \beta \in \Omega \Rightarrow r_\alpha \beta \in \Omega$, then there exists a subset Φ of Ω so that with $B := < \alpha, \beta^\vee >_{\alpha,\beta \in \Phi}$, $\mathcal{D}' := (B, \Phi, \Phi^\vee, V, V^\vee, < \cdot, \cdot >)$ is root data for Ω. In particular $W_\Omega := < r_\alpha \mid \alpha \in \Omega >$ is a Coxeter group.

One observes that in (2) Φ may be infinite even if Π is finite.

Example. If $\Lambda^{25,1}$ is the even unimodular Lorentzian lattice in 26-dimensional space and Σ is the set of vectors of square length 2 in $\Lambda^{25,1}$, then Σ is a root system for suitable root data and Σ has a base in 1-1 correspondence with the Leech lattice. This sets the results of Conway and Sloane into this setting.

(The work described is joint work with A. Pianzola.)

ROOT SYSTEMS OF INFINITE RANK

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We present a structure theory for root systems in arbitrary dimensional spaces or, more generally, 3-graded root systems. It is based on a new type of basis for 3-graded root systems called grid bases. The main results are:

(1) Grid bases always exist (whereas the usual root bases do not exist in general).

(2) Two grid bases of a 3-graded root system are conjugate in the automorphism group of the 3-graded root system.

(3) The reflections with respect to roots from a grid base generate the Weyl group, leading to a presentation of the Weyl group.

(4) Classification of grid bases.

3-graded root systems naturally occur in Lie algebras associated with Jordan pairs via the Kantor-Koecher-Tits construction: a covering grid of a Jordan pair gives rise to a toral subalgebra of the Kantor-Koecher-Tits algebra whose roots form a (in general infinite) 3-graded root system. Idempotents in root spaces, indexed by roots from a grid base, generate the algebra and satisfy 3 simple relations. These generators and relations give a new presentation for the Kantor-Koecher-Tits algebra, i.e. for types A, B, C, D, E_6, and E_7.

ON THE SEMICENTER OF AN ENVELOPING ALGEBRA

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Let L be a finite dimensional Lie algebra over a field k of characteristic zero, $D(L)$ the quotient division ring of the enveloping algebra $U(L)$. It is shown that the weights of the semi-invariants of $D(L)$ form a finitely generated free abelian group $\Lambda_D(L)$. It follows, among other things, that the semicenter $Sz(D(L))$ of $D(L)$ is isomorphic to the group algebra of $\Lambda_D(L)$ over the center $Z(D(L))$. In particular, $Sz(D(L))$ is both factorial and Noetherian. On the other hand, the semicenter $Sz(U(L))$ of $U(L)$ is factorial but not necessarily Noetherian, while the center $Z(U(L))$ does not have to be either one. Next we determine when a localization $U(L)_S$ is primitive, where S is a multiplicatively closed subset of semi-invariants of $U(L)$. In particular, we show that $U(L)_{Z(U(L))}$ is primitive if and only if $Z(D(L))$ is the quotient field of $Z(U(L))$. In that case a more detailed description of $Sz(U(L))$ can be given, while the weights of the semi-invariants of $U(L)$ form a finitely generated, factorial monoid.

(This is joint work with E. Nauwelaerts and P. Wauters.)

REFLECTIONS ON POLYGONS

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Our starting point is the following problem from the 1986 International Mathematics Olympiad.

Given five circularly ordered integers with positive sum, a legal move consists of reversing the sign of a negative entry (if there is one) and then this (positive) number is subtracted from its two neighbors. Show that there cannot be an infinite sequence of legal moves.

If we allow real entries and n of them ($n \geq 3$) instead of five, this becomes a geometric problem involving reflections. The number of legal moves in a maximal sequence is actually the length of a unique element ω in the affine Weyl group $\tilde{A}_{n-1}$. The uniqueness of ω implies that the number of moves and the terminal configuration are independent of the intermediate choices (of negative entries) and depend only on the initial configuration.

We can also give simple formulae for the number of moves and the terminal configuration as a function of the initial configuration using a representation of $\tilde{A}_{n-1}$ due to Lusztig.

There is also a geometric application to convexifying polygons, a question of Kazarinoff related to the isoperimetric problem.

FINITE DIMENSIONAL ALGEBRAS AND ALGEBRAIC GROUPS

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After reviewing the important notion of a triangulated category, we indicate how it leads to the concept of a quasi-hereditary algebra. This notion, due to E. Cline, the speaker, and L. Scott, appears to play an important role in the representation theory of semisimple algebraic groups, Lie algebras, etc. Other important examples of quasi-hereditary algebras arise from consideration of perverse sheaves, poset algebras, etc. We briefly discuss some of these examples, including, in particular, Schur algebras.

THE SCHUBERT CALCULUS AND THE PRINCIPAL HEISENBERG

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I link the classical Schubert calculus, that is the cohomology of Grassmannians, to the representation theory of the principal Heisenberg via the topology of loop groups. In particular, I generalize the Giambelli, Pieri and hook formulas to arbitrary minuscule representations.

CLOSED FORMULAS FOR WEIGHT MULTIPLICITIES OF AFFINE ALGEBRAS

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The values of the weight multiplicities of the integrable irreducible highest weight representations of affine algebras are encoded in its generating function - the Weyl-Kac character formula. Manipulation of this formula led Feingold and Lepowsky to a recursive formula for the multiplicities. In the finite dimensional case the latter is known as Racah's formula.

In the present paper we obtain a closed formula for the weight multiplicities. The formula is given by an infinite series of Rademacher type. Its derivation is the result of bringing together the modular invariance properties of the character due to Kac and Peterson and the tools in analytic number theory introduced by Hardy, Ramanujan, and Rademacher.

The importance of our formula is both theoretical and computational. For example, in the case of the affine algebra of loops in $sl(2, \mathbb{C})$, at level one, the formula reduces to Rademacher's formula for the partition function. This formula offers fairly rapid computation. In general, Racah's formula is suitable for the calculation of the few "top" multiplicities and the remaining ones can be obtained by our formula. Finally, the formula implies asymptotical formulas for the multiplicities.

(This is joint work with C. Moreno.)

ALGEBRAIC GROUPS AND THE REPRESENTATION THEORY OF SOLVABLE GROUPS AND LIE ALGEBRAS

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Hochschild and Mostow developed a theory that associated with a group G (resp. Lie algebra L) the Hopf algebra $\mathcal{R}_k(G)$ (resp. $\mathcal{R}_k(L)$) of *representative functions* with values in a given field k. This commutative, reduced Hopf algebra determines a smooth affine pro-algebraic k-group scheme $\mathcal{G}_k(G)$ (resp. $\mathcal{G}_k(L)$) whose rational representation theory coincides with the finite dimensional representation theory of G (resp. L) over k. Questions about the finite dimensional representation theory of G (resp. L) can then be broken up into two parts, of the sort

(A) Find the kernel of the map $\mathcal{G}_k(H) \longrightarrow \mathcal{G}_k(G)$ induced by the inclusion of a subgroup H of G (similarly for Lie algebras).

(B) Rational representation theory of pro-algebraic affine k-groups.

I present certain results for (A) which, when combined with results of Chevalley, Cline, Parshall, and Scott, etc., for (B), give conditions for extendability of representations of sub-Lie algebras to finite dimensional representations of a Lie algebra, and give results on the faithful representability of solvable groups of finite torsion-free rank.

ENVELOPING ALGEBRAS AND DIVISION RINGS FOR LIE p-ALGEBRAS

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After an elementary proof for the Wedderburn theorem on finite division rings was given to illustrate ideas, the main concern of this talk was the division ring R generated by the enveloping algebra $U(L)$ of a finite dimensional Lie p-algebra L. A subfield K of R was defined to be diagonalizable on R if $R = \sum_\lambda R(\lambda)$ where the λ are functions on K and $R(\lambda) = \{ x \mid [a,x] = \lambda(a)x,\ a \in K \}$. Elementary properties of roots were derived and the question was raised as to whether, given a torus T of L, T can be included in a torus S in R, containing the center, such that the subfield generated by S is both maximal and diagonalizable. It was asserted that this appears to be the case when L is solvable, and it was conjectured to be true in general.

QUERIES IN CHARACTERISTIC p

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A range of questions concerning representations and various universal associative algebras of Lie algebras in characteristic p are discussed. After a review of results on irreducible representations, mainly due to Curtis and to Weisfeiler and Kac, the matter of general, and of indecomposable, representations is taken up. Reference is made to the 1968 results of Pollack on $sl(2)$, and to other ways of producing indecomposables. The recent work of Friedlander-Parshall and of Jantzen on support varieties is briefly introduced.

Sample queries:

(1) Is there a presentation of the irreducible restricted modules for classical algebras like that in the complex case?

(2) Is there a relatively low power of the radical of the restricted universal enveloping algebra such that all indecomposables not annihilated by this power are projective?

INSTABILITY FOR KAC-MOODY GROUPS

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In our Habilitationsschrift (Bonn University, 1984) we stated the following conjecture:

Let G be a Kac-Moody group and $L(\Lambda)$ a highest weight, integrable module of G, $dim L(\Lambda) > 1$. Let $v \in L(\Lambda)$, $v \neq 0$. Then the stabilizer G_v of v in G is contained in a proper parabolic subgroup of G.

As a technical tool to establish this result we proposed a generalization of the Kempf-Rousseau theory of instable vectors in representations of reductive groups to the case of Kac-Moody groups. Such a generalization and a proof of our conjecture was given by Kac-Peterson (Springer LN in Math 1271 (1987)) in the case of a "sufficiently" regular weight Λ. In this talk we give a proof for arbitrary dominant weights. A basic ingredient is the use of Looijenga's partial compactification of a maximal torus T of G (Inventiones Math. 61 (1980)). This compactification has shown up already before in our construction of an adjoint quotient for Kac-Moody groups (cf. Habilitationsschrift, MSRI Vol. 4 (1985) 307-333, as well as our talk at the Workshop on Lie Algebras, Madison, August 1987), and it also determines the Kac-Peterson semigroup completion of G ("Arithmetic and Geometry II", Birkhäuser, Boston (1983)).

THE CLASSIFICATION OF SIMPLE MODULAR LIE ALGEBRAS I, II

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I. We present a theory of p-envelopes of Lie algebras:

Definition. A triple $(G, [p], i)$ consisting of a restricted Lie algebra $(G, [p])$ and a homomorphism $i : L \longrightarrow G$ is called a *p – envelope* of L if (a) i is injective, and (b) the p-algebra generated by $i(L)$ equals G.

Theorem 1. Let L be finite dimensional. Then
(1) L has a finite dimensional p-envelope.
(2) Let $(G_k, [p]_k, i_k)$ be two p-envelopes. Then there exists a nonrestricted homomorphism $\phi : G_1 \longrightarrow G_2$ and a subspace $J \subset C(G_2)$ in the center of G_2 such that

$$G_2 = \phi(G_1) \oplus J, \quad \phi o i_1 = i_2, \quad ker\phi \subset C(G_1).$$

As applications, a conceptual proof of Iwasawa's theorem and a general modular version of the Dixmier-Blattner result are presented.

II. **Definition.** Let L be a Lie algebra and $(L_p, [p], i)$ be a p-envelope of L. Suppose that H is a subalgebra of L and H_p the p-subalgebra of L_p generated by $i(H)$. Then

$$TR(H,L) := max\,\{\,dimT \mid T \text{ is a torus of } H_p + C(L_p)/C(L_p)\}$$

is called the *absolute toral rank of H in L.*
This definition comprises several different notions of rank.

Definition. A torus T in L_p is called *standard with respect to L* if $C_L(T)^{(1)}$ acts nilpotently on L.

Main Theorem 2. Let L be a simple finite dimensional Lie algebra over an algebraically closed field of characteristic $p > 7$. Suppose that T is a torus in some p-envelope L_p of L with maximal absolute toral rank. Then T is standard with respect to L_p.

Corollary 3. If $TR(L, L) \leq 2$, then L is classical or of Cartan type.

III. A (fairly detailed) list of possible 2-sections with respect to a torus of maximal absolute toral rank is given. As a consequence a construction due to Benkart-Osborn-Strade yields a subalgebra whose relevance for the classification problem of simple Lie algebras is discussed.

Theorem 4. Assume that L and T are as in the main theorem. If $C_L(T)$ is a Cartan subalgebra of L, then L is classical or of Cartan type.

TABLEAUX IN THE REPRESENTATION THEORY OF THE CLASSICAL LIE GROUPS

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Partitions and tableaux were introduced early in the representation theory of the symmetric group by Alfred Young. Following work of Specht, Weyl used Young symmetrisers (operators associated to standard tableaux) to construct the irreducible modules for the polynomial representations of $GL(n)$, $O(n)$ and $Sp(2n)$.

More recent work by mathematical physicists such as R.C. King addresses the question of finding sets of tableaux which index the weights of a given irreducible representation, so that a weighted generating function of these tableaux is precisely the formal character of the representation.

We describe these tableaux for each of the classical Lie groups G, and discuss combinatorial ways to approach the problem of computing tensor space decompositions. In the $GL(n)$-case the combinatorial algorithm is the well-known Robinson-Schensted insertion scheme. Analogous insertion schemes for $Sp(2n)$ and $SO(2n+1)$ are described.

We also discuss the use of combinatorial tools to compute multiplicities in the decomposition of a tensor product of two irreducible G-modules, and to compute the decompositions $GL(2n) \downarrow Sp(2n)$, $GL(m) \downarrow O(m)$.

MODULAR SUBALGEBRAS IN LIE ALGEBRAS OF PRIME CHARACTERISTIC

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Let L be a finite dimensional Lie algebra over a perfect field F of characteristic $p \neq 2, 3$. A subalgebra M of L is called modular in L if it is a modular element in the lattice of subalgebras of L.

We are concerned with the effect the modular subalgebras have on the structure of L. For $char(F) = 0$, this was done by Amayo and Schwarz (1980).

Assume that $M \leq L$ is modular. Let M_L denote the largest ideal of L contained in M. For $dim\, M/M_L \leq 2$, we obtain the same results as Amayo and Schwarz. If $char(F) = 0$, all possibilites for modular subalgebras end here.

Suppose then that $char(F) = p \neq 0$. Question: Are the Zassenhaus algebras, $Z_n(F)$, the only Lie algebras having a modular subalgebra of dimension > 2? We get an affirmative answer in some cases.

Let $L = Z_n(F)$. We see that L has a unique modular subalgebra L_0. We find conditions to characterize the pair (L_0, L).

Now assume that F is algebraically closed. Then we obtain the following:

Theorem. Let L be a simple Lie algebra of Cartan type and L_0 denote the standard maximal subalgebra of L. Then, no maximal subalgebra $M \neq L_0$ is modular in L (except perhaps for $L = K(3 : \underline{n} : \Phi)^{(2)}$ and $\Phi \neq 1$).

Corollary 1. The algebras $W(1 : \underline{n})$ are the only graded simple Lie algebras of Cartan type having a modular and maximal subalgebra.

Corollary 2. If L is a restricted simple algebra having a modular and maximal subalgebra, then $L \cong sl(2)$ or $W(1 : \underline{1})$.

Finally, we consider the algebras of type $H(2 : \underline{n} : \Phi)^{(2)}$.

DETERMINATION OF THE EXPONENTIAL MAP FOR HOLOMORPHS OF COMPLEX LIE GROUPS, AND A CRITERION FOR FRACTIONAL ITERATIONS OF FORMAL AUTOMORPHISMS

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The code "formal automorphisms" refers to algebra automorphisms of the formal power series $\overline{R} = \overline{R}^{(n)}$ over $\mathbb{C}$ in variables $X_1, X_2, ..., X_n$. We give a necessary and sufficient condition for a formal automorphism A to be of the form e^D for a unique derivation D with prescribed linear part. This is done by reducing the problem to the determination of (the image of) the exponential map $\mathcal{G}_d \longrightarrow G_d$ for a family of complex Lie groups G_d with Lie algebra $\mathcal{G}_d$. In fact, G_d is the holomorph (semidirect extension) of G_d by (the additive group of) its representation space M_d consisting of all linear maps from $V = \mathbb{C}$-span of $X_1, X_2, ..., X_n$ to $Sym^d V$ spanned by dth-degree monomials in those; the relevance of M_d comes from the fact that the "dth-degree part" of A may be viewed as an element of M_d.

More generally, for any real or complex Lie group H and H-module M we give an explicit description of the map $exp : \mathcal{G} = LieG \longrightarrow G$ for the holomorph $G = G_{H,M}$ of H by M. If $(Y, v) \in LieH \ltimes M = \mathcal{G}$ is any element of the Lie algebra of G, its image under exp equals $(e^Y, \tilde{Y} \cdot v)$ where $\tilde{Y} = Id + \rho(Y)/2! + \rho(Y)^2/3! + \ldots = (e^{\rho(Y)} - 1)/\rho(Y)$ (formally), $\rho(Y)$ being the infinitesimal action of Y on M. When $exp : LieH \longrightarrow H$ is surjective, this gives a transparent understanding of the failure of $exp : LieG \longrightarrow G$ from being surjective. The desired criterion for $A = e^D$ above then readily follows, which is essentially a coordinate-free clean version of a result of Ludwig Reich (1977) in terms if conjugability of A to a certain kind of normal form. Our result implies known sufficient conditions from Shlomo Sternberg (1961), that date back to Daniel Lewis (1939).

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