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# Probability Theory and its Applications in China 

# CONTEMPORARY Mathematics 

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# Probability Theory and its Applications in China 

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## Preface

Probability theory has always been an active mathematical field in China, and, until very recently, almost all of its research results were written in Chinese. This book consists essentially of surveys of probability theory research in China written by Chinese probabilists. We intended to collect as complete a volume as possible about the contributions and progresses of theoretical and applied probability theory research in China. Our object in the present volume is twofold. First, we have tried to give an account of the most significant accomplishments obtained by the Chinese probabilists through the past few decades, focusing on the timely reporting of the research achievements in the past ten years. Second, we tried to promote academic exchange and a better understanding between the communities of probability theory researchers in China and the rest of the world. Finally, we hope the abstract and explicit mathematical results presented here will soon be found useful in practical investigations through joint research projects between Eastern and Western probabilists. All the papers are considered to be in their final forms and will not be submitted for publications elsewhere.

The content of the book will reflect a fairly complete coverage of the research activities of the theoretical nature of probability theory. Only selective topics of applied aspects are covered. Nevertheless, we believe strongly that this book will enhance communications between the East and West in jointly advancing probability theory. Therefore, it is our great pleasure to be able to present such a book to the readers outside of China. Due to the limitation of our ability, time, energy, and various other reasons, we regret the inevitable omissions of some important works of several outstanding Chinese colleagues. However, some of their research works have already been reported in Western languages, such as the work of Professors Wang Zi-Kun and Yang Xiang-Qun in Markov chains and of Professor Guo Mao-Zheng in the hydrodynamic limit and etc.; some works in applications of the probability theory, such as queues for service, will be included in other volumes. We also have purposely left out some fine works by many relatively young but promising researchers, but we are confident that they will be able to achieve more penetrating and systematic results in the near future. Thus it is appropriate to wait to report these new developments in the future.

As editors, we are indebted to the contributors of this book for their cooperation in the preparation of the manuscripts. We want to express our thanks to Professor Robert M. Fossum, the Secretary of AMS, for his encouragement and
interest in the proposal，to Professor Dan Burns，the past chair of the Contem－ porary Mathematics Series，for overseeing the project，and the American Math－ ematical Society，especially Ms．Mary Lane，the former Director of Publication Division，Ms．Donna L．Harmon，the Assistant to the Director of Publication， and Ms．Alison Buckser，the Production Editor of this volume，for its endorse－ ment of the project and technical assistance in preparing camera－ready copy of the manuscripts．In addition，we want to acknowledge the support from the National Natural Science Foundation of China，since，otherwise，many of the research accomplishments reported here would be impossible to obtain．We also want to thank Professor Wang Yuan for his encouragement and concern for the project，and to Professor Liu Shi－Heng for reading over most of the manuscripts and improving the presentations．Finally，we would like to thank the group of graduate students of probability theory led by Mr．Li Yong at Beijing Normal University，for their efforts in using $\mathcal{A} \mathcal{M} \mathcal{S}-\mathrm{T}_{\mathrm{E}} \mathrm{X}$ to make the book more appealing．

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