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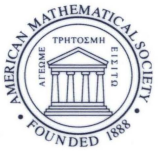

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Inverse Scattering and Applications

CONTEMPORARY MATHEMATICS

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Inverse Scattering and Applications

Proceedings of a Conference
on Inverse Scattering on the Line
held June 7–13, 1990
at the University of Massachusetts, Amherst
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and the Office of Naval Research

D. H. Sattinger
C. A. Tracy
S. Venakides
Editors

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Preface

This conference covered a variety of topics in inverse problems: inverse scattering problems on the line; inverse problems in higher dimensions; inverse conductivity problems; and numerical methods. In addition, problems from statistical physics were covered, including monodromy problems, quantum inverse scattering, and the Bethe ansatz. One of the aims of the conference was to bring together researchers in a variety of areas of inverse problems. All of these areas have seen intensive activity in recent years.

Inverse conductivity problems

This class of problems was discussed by David Isaacson and Margaret Cheney of Renssalaer Polytechnic Institute and by Gunther Uhlmann of the University of Washington. Uhlmann discussed his work with John Sylvester on the problem of determining anisotropic conductivities in a region from measurements made on the boundary. These measurements may include the Dirichlet-Neumann map or knowledge of the geodesics. Margaret Cheney discussed various algorithms for reconstructing the conductivities from the data: these included iterative methods, and Calderon's methods. David Isaacson discussed experimental work being carried out at Renssalaer Polytechnic Institute and ended his talk with an intriguing videotape of actual inverse imaging experiments on a human subject (himself).

Adrian Nachman, of the University of Rochester, gave an overview of inverse scattering and conductivity problems. Joyce McLaughlin, of Renssalaer Polytechnic Institute, presented recent results on inverse spectral problems for second order differential operators.

Numerical methods

Vladimir Rokhlin of Yale University described a numerical algorithm for inverse scattering based on a Riccati equation for the impedence function combined with certain trace formulae for the unknown functions. Numerical experiments performed in one dimension have shown themselves to be stable, rigorous, and extremely efficient. He hopes to be able to extend the methods to two and three dimensional problems.

Soliton problems

One dimensional inverse scattering methods are a fundamental tool in the theory of completely integrable systems. Percy Deift of the Courant Institute opened the

conference with a beautiful summary of the theory of inverse scattering for nth order ordinary differential operators. Thanks to recent work by Xin Zhou and Deift, this theory is now complete. Thomas Kappeler of Brown University discussed action angle variables for the periodic KdV equation. Richard Beals of Yale University spoke on his recent work with D. Sattinger on action angle variables for integrable systems based on first order $n \times n$ isospectral operators. The construction of action angle variables for these infinite dimensional completely integrable systems is based on the scattering transform.

Scattering theory was also used by Bjorn Birnir of University of California, Santa Barbara and S. Kichenessamy of the Courant Institute in their (independent) work showing that only the Sine-Gordon equation can support breather solutions.

M. Wickerhauser of the University of Georgia reported on joint work with R. Coifman of Yale University on some of the special problems of the scattering transform for the Benjamin-Ono equation. Their work gives estimates for some previously formal work associated with the Benjamin-Ono hierarchy.

S. Venakides of Duke University reported on joint work with P. Deift of the Courant Institute and R. Oba of Tulane University on the Toda Shock problem. Long time asymptotic analysis of the explicit solution is carried out by the inverse scattering method. Residual oscillations are derived and analyzed when the initial velocity exceeds a critical value. The results are in agreement with earlier numerical experiments by Straub and Holian, and Flaschka and McLaughlin.

David McLaughlin of Princeton University discussed chaos and heteroclinic orbits of perturbed integrable systems.

Three dimensional problems

A. Ramm of Kansas State University and T. Aktosun of the University of Texas at Dallas presented their work on three dimensional problems. Ramm talked about the C Property and Aktosun talked on the Wiener-Hopf factorization of the scattering operator in three dimensions, based on ideas of R. Newton.

Statistical physics

A number of problems in statistical physics lead to problems involving inverse monodromy or inverse scattering, and several of the talks addressed these areas. V. Korepin, of the University of New York at Stonybrook, discussed correlation functions for the quantized version of the nonlinear Schrödinger equation. In many cases, the correlation functions satisfy nonlinear differential equations of Painlevé type. The Painlevé equations, in turn, are associated in a direct way with certain monodromy problems; in fact, the monodromy problems play a role analogous to the isospectral operators in the theory of completely integrable systems. Inverse monodromy problems thus play an important role. John Palmer of the University of Arizona talked about the Cauchy Riemann operators associated with such inverse monodromy problems and their infinite dimensional determinants as tau functions

for the problem. The tau functions are in fact the partition function of statistical mechanics. Hank Thacker of the University of Virginia talked about related topics including spin chains and vertex models. Craig Tracy spoke on monodromy problems in higher dimensions, specifically some isomonodromy problems for the Laplacian on the Poincaré disk. The two point correlation function can be expressed in terms of Painlevé VI.

During the course of the conference, Persi Diaconis, who was attending the other conference at Amherst, overheard mention of the "Bethe ansatz" during an informal discussion at coffee break. It developed that there was a connection between the order/disorder transitions in "card shuffling" problems that Diaconis has been working on, and the Bethe ansatz method used in connection with the statistical problems being discussed by Korepin and Thacker. Diaconis agreed to give a special lecture, at 8:30 a.m. Sunday morning, on his work on order/disorder transitions. Several discussions resulted, and a round table session took place on Monday evening to understand the relationships.

D. H. Sattinger

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