

CONTEMPORARY MATHEMATICS

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Abelian Groups and Noncommutative Rings

A Collection of Papers in Memory of
Robert B. Warfield, Jr.



American Mathematical Society

Abelian Groups and Noncommutative Rings

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Robert B. Warfield, Jr.

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L. Fuchs
K. R. Goodearl
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Editors



American Mathematical Society
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Preface

This collection of research papers has been written in memory of Robert B. Warfield, Jr. His untimely death was a severe blow to his mathematical friends, colleagues and students, who conceived this volume as a tribute to him.

The diversity of the subject matter covered here reflects Warfield's own wide-ranging interests. Roughly speaking, the papers may be divided into abelian group theory and ring theory, the main focal points of Warfield's research. More information on his contributions to mathematics can be found in the two survey articles "The abelian group papers of Robert B. Warfield, Jr." and "Warfield in ring theory".

The editors wish to express their particular appreciation to the mathematicians who acted as referees for the contributions submitted for this volume. We would also like to thank Katya Miličić, who converted many of the individual papers into $\text{T}_{\text{E}}\text{X}$ for us, and Dragan Miličić for his $\text{T}_{\text{E}}\text{X}$ nical assistance.

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L. Fuchs, K. R. Goodearl, J. T. Stafford,
and C. Vinsonhaler, Editors

This collection of research papers is dedicated to the memory of the distinguished algebraist Robert B. Warfield, Jr. Focusing on abelian group theory and noncommutative ring theory, the book covers a diversity of topics reflecting Warfield's wide-ranging interests and includes two articles surveying his contributions to mathematics. Because the articles have been refereed to high standards and will not appear elsewhere, this volume is indispensable to any researcher in noncommutative ring theory or abelian group theory. With papers by some of the major leaders in the field, this book will also be important to anyone interested in these areas, as it provides an overview of current research directions.

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