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Mathematical Aspects of Classical Field Theory



American Mathematical Society

Mathematical Aspects of Classical Field Theory

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Mathematical Aspects of Classical Field Theory

Proceedings of the AMS-IMS-SIAM
Joint Summer Research Conference
held July 20–26, 1991,
with support from
the National Science Foundation

Mark J. Gotay
Jerrod E. Marsden
Vincent Moncrief



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Preface

Classical field theory is characterized by its examples, old and new: general relativity, electromagnetism, Yang-Mills theory, strings, super theories, matter theories (fluids, plasmas, elasticity), topological field theories, and so forth. The subject has undergone a revitalization in the last few years with the infusion of ideas from symplectic and Poisson geometry, integrable systems, and the geometry behind the variational calculus, to mention just some of the driving forces. The reciprocity between quantum systems and their classical counterparts has also played an important role, recent developments in BRST theory being a case in point. This conference was designed to bring together experts in these aspects of classical field theory to exchange ideas on exciting recent developments.

The conference was well attended, with 81 participants from 26 states and provinces and 16 foreign countries. The lectures at the conference divided roughly into the categories of:

1. The calculus of variations (the variational bicomplex, higher order theory, multisymplectic structures),
2. BRST theory,
3. Geometric methods (symplectic and Poisson techniques, momentum mappings, Dirac constraint theory, integrable systems, geometric quantization),
4. Global analysis and general relativity (cosmic censorship, initial value problem, quantum gravity).

We are grateful to the American Mathematical Society for their organizational and publishing skills that aided the conference considerably. In particular, we would like to thank Carole Kohanski and Donna Harmon for their efforts.

We would also like to thank Christian Duval, Lucas Hsu, Johannes Huebschmann, and Jim Isenberg for their very able help in organizing the special sessions for contributed talks.

Mark Gotay, Jerrold Marsden, and Vince Moncrief

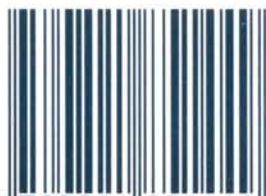
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