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Vinay Deodhar



Kazhdan-Lusztig Theory and Related Topics

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139

Kazhdan-Lusztig Theory and Related Topics

Proceedings of an AMS Special Session
held May 19–20, 1989
at the University of Chicago,
Lake Shore Campus, Chicago, Illinois

Vinay Deodhar



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Preface

In the years 1979–80, Kazhdan and Lusztig published two papers which dealt with interesting problems from different topics related to Lie Theory such as structure of Verma modules and its relation with the combinatorics of Bruhat ordering in Weyl groups, geometry of Schubert varieties in generalized flag manifolds, and primitive ideals in enveloping algebras. As far as these individual topics are concerned, some noteworthy results had been obtained by mathematicians prior to the work of Kazhdan and Lusztig. However, the link between these seemingly diverse topics was made clear in the papers of Kazhdan–Lusztig. In fact, these papers seem to have started a new topic in Lie Theory and related areas, a topic which can be called Kazhdan–Lusztig Theory. Since then a number of mathematicians have worked on problems related to this circle of ideas and several interesting results have been obtained. The aim of this special volume is to put together articles which deal with new results on different aspects of the Kazhdan–Lusztig Theory and related topics.

This volume has its origin in the special session on this topic at the A.M.S. meeting in Chicago in May 1989. Several mathematicians could not attend this meeting due to conflicts with conferences held elsewhere in the U.S.A. and Europe. Considering the enthusiasm with which this session was received by the audience, it seemed natural to put together a volume based on the proceedings of the special session. In addition to the speakers, several mathematicians were invited to contribute articles. The response was very encouraging. It has been a very rewarding experience for me to edit this volume.

My sincere thanks are due to the authors for their contributions to the volume and their patience with me through several schedule changes, to the referees for their time and careful evaluation of the papers, and to the American Mathematical Society for publishing this volume.

Vinay Deodhar
Bloomington
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