

CONTEMPORARY MATHEMATICS

143

A Tribute to Emil Grosswald: Number Theory and Related Analysis



Recent Titles in This Series

- 143 **Marvin Knopp and Mark Sheingorn, Editors**, A tribute to Emil Grosswald: Number theory and related analysis, 1993
- 142 **Chung-Chun Yang and Sheng Gong, Editors**, Several complex variables in China, 1993
- 141 **A. Y. Cheer and C. P. van Dam, Editors**, Fluid dynamics in biology, 1993
- 140 **Eric L. Grinberg, Editor**, Geometric analysis, 1992
- 139 **Vinay Deodhar, Editor**, Kazhdan-Lusztig theory and related topics, 1992
- 138 **Donald St. P. Richards, Editor**, Hypergeometric functions on domains of positivity, Jack polynomials, and applications, 1992
- 137 **Alexander Nagel and Edgar Lee Stout, Editors**, The Madison symposium on complex analysis, 1992
- 136 **Ron Donagi, Editor**, Curves, Jacobians, and Abelian varieties, 1992
- 135 **Peter Walters, Editor**, Symbolic dynamics and its applications, 1992
- 134 **Murray Gerstenhaber and Jim Stasheff, Editors**, Deformation theory and quantum groups with applications to mathematical physics, 1992
- 133 **Alan Adolphson, Steven Sperber, and Marvin Tretkoff, Editors**, p -Adic methods in number theory and algebraic geometry, 1992
- 132 **Mark Gotay, Jerrold Marsden, and Vincent Moncrief, Editors**, Mathematical aspects of classical field theory, 1992
- 131 **L. A. Bokut', Yu. L. Ershov, and A. I. Kostrikin, Editors**, Proceedings of the International Conference on Algebra Dedicated to the Memory of A. I. Mal'cev, Parts 1, 2, and 3, 1992
- 130 **L. Fuchs, K. R. Goodearl, J. T. Stafford, and C. Vinsonhaler, Editors**, Abelian groups and noncommutative rings, 1992
- 129 **John R. Graef and Jack K. Hale, Editors**, Oscillation and dynamics in delay equations, 1992
- 128 **Ridgley Lange and Shengwang Wang**, New approaches in spectral decomposition, 1992
- 127 **Vladimir Olikier and Andrejs Treibergs, Editors**, Geometry and nonlinear partial differential equations, 1992
- 126 **R. Keith Dennis, Claudio Pedrini, and Michael R. Stein, Editors**, Algebraic K -theory, commutative algebra, and algebraic geometry, 1992
- 125 **F. Thomas Bruss, Thomas S. Ferguson, and Stephen M. Samuels, Editors**, Strategies for sequential search and selection in real time, 1992
- 124 **Darrell Haile and James Osterburg, Editors**, Azumaya algebras, actions, and modules, 1992
- 123 **Steven L. Kleiman and Anders Thorup, Editors**, Enumerative algebraic geometry, 1991
- 122 **D. H. Sattinger, C. A. Tracy, and S. Venakides, Editors**, Inverse scattering and applications, 1991
- 121 **Alex J. Feingold, Igor B. Frenkel, and John F. X. Ries**, Spinor construction of vertex operator algebras, triality, and $E_8^{(1)}$, 1991
- 120 **Robert S. Doran, Editor**, Selfadjoint and nonselfadjoint operator algebras and operator theory, 1991
- 119 **Robert A. Melter, Azriel Rosenfeld, and Prabir Bhattacharya, Editors**, Vision geometry, 1991
- 118 **Yan Shi-Jian, Wang Jiagang, and Yang Chung-chun, Editors**, Probability theory and its applications in China, 1991
- 117 **Morton Brown, Editor**, Continuum theory and dynamical systems, 1991
- 116 **Brian Harbourne and Robert Speiser, Editors**, Algebraic geometry: Sundance 1988, 1991
- 115 **Nancy Flournoy and Robert K. Tsutakawa, Editors**, Statistical multiple integration, 1991
- 114 **Jeffrey C. Lagarias and Michael J. Todd, Editors**, Mathematical developments arising from linear programming, 1990

(Continued in the back of this publication)



Photograph courtesy of C. J. Mozzochi.

Emil Grosswald

Photograph taken by Dr. C. J. Mozzochi,
July, 1987 at Laval University, Quebec.

CONTEMPORARY MATHEMATICS

143

A Tribute to Emil Grosswald: Number Theory and Related Analysis

Marvin Knopp
Mark Sheingorn
Editors



EDITORIAL BOARD

Richard W. Beals, managing editor
Craig Huneke Linda Preiss Rothschild
Clark Robinson Peter Winkler

1991 *Mathematics Subject Classification*. Primary 05A16, 05A30, 11-06, 14H45, 33C05.

Library of Congress Cataloging-in-Publication Data

A Tribute to Emil Grosswald: number theory and related analysis/Marvin Knopp, Mark Sheingorn, editors.

p. cm. - (Contemporary mathematics, ISSN 0271-4132; 143)

ISBN 0-8218-5155-1

1. Number theory. 2. Combinatorial analysis. I. Grosswald, Emil. II. Knopp, Marvin Isadore, 1933-. III. Sheingorn, Mark, 1944-. IV. Series: Contemporary mathematics (American Mathematical Society); v. 143.

QA241.T75 1993

512'.7-dc20

92-39436

CIP

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Manager of Editorial Services, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248.

The appearance of the code on the first page of an article in this book indicates the copyright owner's consent for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that the fee of \$1.00 plus \$.25 per page for each copy be paid directly to the Copyright Clearance Center, Inc., 27 Congress Street, Salem, Massachusetts 01970. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

Copyright ©1993 by the American Mathematical Society. All rights reserved.

The American Mathematical Society retains all rights except those granted to the United States Government.

Printed in the United States of America.

The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability. ∞

Most of the articles in this volume were printed directly from copy prepared by the authors.

Some articles were prepared using *AMS-TEX*, the American Mathematical Society's *TEX* macro system.

10 9 8 7 6 5 4 3 2 1 98 97 96 95 94 93

Contents

Preface	viii
In appreciation of Emil Grosswald	1
Ph.D. students of Emil Grosswald	9
Publications of Emil Grosswald	11
A rather exact formula for the number of plane partitions GERT ALMKVIST	21
On Ramanujan's empirical calculation for the Rogers-Ramanujan identities GEORGE E. ANDREWS	27
Integers expressible in a given number of ways as a sum of two squares PAUL T. BATEMAN	37
A theorem of Ramanujan on certain alternating series BRUCE C. BERNDT AND JAMES LEE HAFNER	47
Combinatorial equivalence of definitions of the Schur function DAVID M. BRESSOUD AND SHI-YUAN WEI	59
Is there life on finite upper half planes? NANCY CELNIKER, STEVEN POULOS, AUDREY TERRAS, CINDY TRIMBLE, AND ELINOR VELASQUEZ	65
Rational period functions for $PSL(2, \mathbb{Z})$ YJ. CHOIE AND D. ZAGIER	89
Rational period functions and indefinite binary quadratic forms, III L. ALAYNE PARSON	109
Hypergeometric and modular function identities, and new rational approx- imations to and continued fraction expansions of classical constants and functions D. V. CHUDNOVSKY AND G. V. CHUDNOVSKY	117
Orbital modular equations HARVEY COHN	163

A mean-value theorem for class numbers of quadratic extensions BORIS A. DATSKOVSKY	179
On a generalization of Farey sequences, I FRANCINE DELMER AND JEAN-MARC DESHOUILERS	243
Sieve auxiliary functions II HAROLD G. DIAMOND, H. HALBERSTAM, AND H.-E. RICHERT	247
A relation between cubic exponential and Kloosterman sums W. DUKE AND H. IWANIEC	255
Function theory for Rogers-Ramanujan-like partition identities LEON EHRENPREIS	259
Forcing two sums simultaneously P. ERDÖS, D. J. NEWMAN, AND J. KNAPPENBERGER	321
Difference polynomials RONALD J. EVANS, KENNETH B. STOLARSKY, AND JOHN J. WAVRIK	329
An application of Ehrenpreis's basis method to the Rogers-Ramanujan identities JANE E. FRIEDMAN	347
Extensions of some extremal properties of prime divisors to Poisson limit theorems JANOS GALAMBOS	363
Rational period functions with irrational poles are not Hecke eigenfunctions ELLEN GETHNER	371
On the number of Fourier coefficients that determine a modular form DORIAN GOLDFELD AND JEFFREY HOFFSTEIN	385
An explicit formula of Landau and its applications to the theory of the zeta-function S. M. GONEK	395
Multiplicative properties of η -products II BASIL GORDON AND KIM HUGHES	415
Counting subgroups of given index in Hecke groups MICHAEL GRADY AND MORRIS NEWMAN	431
Relatively prime values of polynomials JAMES LEE HAFNER, PETER SARNAK, AND KEVIN MCCURLEY	437
A new proof that every odd triperfect number has at least twelve prime factors PETER HAGIS, JR.	445

A Hecke-Weil correspondence theorem for automorphic integrals on $\Gamma_0(N)$, with arbitrary rational period functions JOHN H. HAWKINS AND MARVIN I. KNOPP	451
Lagrange's theorem for Hecke triangle groups J. LEHNER	477
Base change and the Birch-Swinnerton-Dyer conjecture M. RAM MURTY AND V. KUMAR MURTY	481
A "natural" proof of the non-vanishing of L-series D. J. NEWMAN	495
Nonexistence of Siegel zeros in towers of radical extensions ANDREW M. ODLYZKO AND CHRIS M. SKINNER	499
Modular integrals and indefinite binary quadratic forms L. ALAYNE PARSON	513
Diagonalizing Eisenstein series, II ROBERT A. RANKIN	525
Multiplier systems for the Hecke groups $G(\sqrt{2})$ and $G(\sqrt{3})$ DAVID ROSEN	539
Low height Hecke triangle group geodesics MARK SHEINGORN	545
On the Shimura lift for Hilbert modular forms THOMAS R. SHEMANSKE AND LYNNE H. WALLING	561
Dirichlet's class-number formula revisited H. M. STARK	571
Closed form (pun intended!) DORON ZEILBERGER	579
Gert Almkvist's generalization of a mistake of Bourbaki DORON ZEILBERGER	609

PREFACE

The papers in this volume are as diverse as were Emil Grosswald's interests. For this reason, and because of the frequent difficulty in categorizing an article, we have chosen to present the works alphabetically, by author. There is a single exception to this: one of the articles of Parson, "Rational Period Functions and Binary Quadratic Forms, III", follows immediately after that of Choie and Zagier, "Rational Period Functions for $PSL(2, \mathbb{Z})$ ". We chose this arrangement because the two articles prove the same result, independently and by somewhat different methods.

Marvin Knopp and Mark Sheingorn

Recent Titles in This Series

(Continued from the front of this publication)

- 113 **Eric Grinberg and Eric Todd Quinto, Editors**, Integral geometry and tomography, 1990
- 112 **Philip J. Brown and Wayne A. Fuller, Editors**, Statistical analysis of measurement error models and applications, 1990
- 111 **Earl S. Kramer and Spyros S. Magliveras, Editors**, Finite geometries and combinatorial designs, 1990
- 110 **Georgia Benkart and J. Marshall Osborn, Editors**, Lie algebras and related topics, 1990
- 109 **Benjamin Fine, Anthony Gaglione, and Francis C. Y. Tang, Editors**, Combinatorial group theory, 1990
- 108 **Melvyn S. Berger, Editor**, Mathematics of nonlinear science, 1990
- 107 **Mario Milman and Tomas Schonbek, Editors**, Harmonic analysis and partial differential equations, 1990
- 106 **Wilfried Sieg, Editor**, Logic and computation, 1990
- 105 **Jerome Kaminker, Editor**, Geometric and topological invariants of elliptic operators, 1990
- 104 **Michael Makkai and Robert Paré**, Accessible categories: The foundations of categorical model theory, 1989
- 103 **Steve Fisk**, Coloring theories, 1989
- 102 **Stephen McAdam**, Primes associated to an ideal, 1989
- 101 **S.-Y. Cheng, H. Choi, and Robert E. Greene, Editors**, Recent developments in geometry, 1989
- 100 **W. Brent Lindquist, Editor**, Current progress in hyperbolic systems: Riemann problems and computations, 1989
- 99 **Basil Nicolaenko, Ciprian Foias, and Roger Temam, Editors**, The connection between infinite dimensional and finite dimensional dynamical systems, 1989
- 98 **Kenneth Appel and Wolfgang Haken**, Every planar map is four colorable, 1989
- 97 **J. E. Marsden, P. S. Krishnaprasad, and J. C. Simo, Editors**, Dynamics and control of multibody systems, 1989
- 96 **Mark Mahowald and Stewart Priddy, Editors**, Algebraic topology, 1989
- 95 **Joel V. Brawley and George E. Schnibben**, Infinite algebraic extensions of finite fields, 1989
- 94 **R. Daniel Mauldin, R. M. Shortt, and Cesar E. Silva, Editors**, Measure and measurable dynamics, 1989
- 93 **M. Isaacs, A. Lichtman, D. Passman, S. Sehgal, N. J. A. Sloane, and H. Zassenhaus, Editors**, Representation theory, group rings, and coding theory, 1989
- 92 **John W. Gray and Andre Scedrov, Editors**, Categories in computer science and logic, 1989
- 91 **David Colella, Editor**, Commutative harmonic analysis, 1989
- 90 **Richard Randell, Editor**, Singularities, 1989
- 89 **R. Bruce Richter, Editor**, Graphs and algorithms, 1989
- 88 **R. Fossum, W. Haboush, M. Hochster, and V. Lakshmibai, Editors**, Invariant theory, 1989
- 87 **Laszlo Fuchs, Rüdiger Göbel, and Phillip Schultz, Editors**, Abelian group theory, 1989
- 86 **J. Ritter, Editor**, Representation theory and number theory in connection with the local Langlands conjecture, 1989
- 85 **Bor-Luh Lin, Editor**, Banach space theory, 1989
- 84 **Stevo Todorcevic**, Partition problems in topology, 1989

(See the AMS catalog for earlier titles)

**A Tribute to Emil Grosswald:
Number Theory and Related Analysis**
Marvin Knopp and Mark Sheingorn, Editors

Emil Grosswald was a mathematician of great accomplishment and remarkable breadth of vision. This volume pays tribute to the span of his mathematical interests, which is reflected in the wide range of papers collected here. With contributions by some of the leading contemporary researchers in number theory, modular functions, combinatorics, and related analysis, this book will be of interest to graduate students and specialists in these fields. The high quality of the articles and their close connection to current research trends make this volume a must for any mathematics library.

ISBN 0-8218-5155-1



9 780821 851555