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A Tribute to Emil Grosswald: Number Theory and Related Analysis

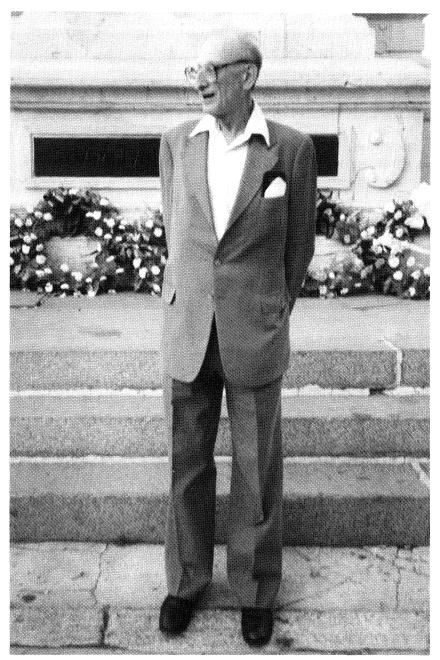


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CONTEMPORARY MATHEMATICS

143

A Tribute to Emil Grosswald: Number Theory and Related Analysis

Marvin Knopp Mark Sheingorn Editors



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1991 Mathematics Subject Classification. Primary 05A16, 05A30, 11-06, 14H45, 33C05.

Library of Congress Cataloging-in-Publication Data

A Tribute to Emil Grosswald: number theory and related analysis/Marvin Knopp, Mark Sheingorn, editors.

p. cm. - (Contemporary mathematics, ISSN 0271-4132; 143) ISBN 0-8218-5155-1

1. Number theory. 2. Combinatorial analysis. I. Grosswald, Emil. II. Knopp, Marvin Isadore, 1933–. III. Sheingorn, Mark, 1944–. IV. Series: Contemporary mathematics (American Mathematical Society); v. 143. QA241.T75 1993 512'.7-dc20 92-39436

92-39436 CIP

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the American Mathematical Society's TEX macro system.

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PREFACE

The papers in this volume are as diverse as were Emil Grosswald's interests. For this reason, and because of the frequent difficulty in categorizing an article, we have chosen to present the works alphabetically, by author. There is a single exception to this: one of the articles of Parson, "Rational Period Functions and Binary Quadratic Forms, III", follows immediately after that of Choie and Zagier, "Rational Period Functions for $PSL(2, \mathbb{Z})$ ". We chose this arrangement because the two articles prove the same result, independently and by somewhat different methods.

Marvin Knopp and Mark Sheingorn

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A Tribute to Emil Grosswald: Number Theory and Related Analysis Marvin Knopp and Mark Sheingorn, Editors

Emil Grosswald was a mathematician of great accomplishment and remarkable breadth of vision. This volume pays tribute to the span of his mathematical interests, which is reflected in the wide range of papers collected here. With contributions by some of the leading contemporary researchers in number theory, modular functions, combinatorics, and related analysis, this book will be of interest to graduate students and specialists in these fields. The high quality of the articles and their close connection to current research trends make this volume a must for any mathematics library.

