

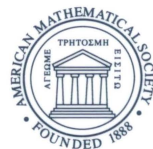
CONTEMPORARY MATHEMATICS

147

Graph Structure Theory

Proceedings of a Joint Summer
Research Conference on Graph Minors
held June 22 to July 5, 1991, at the
University of Washington, Seattle

Neil Robertson
Paul Seymour
Editors



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Preface

This volume contains the proceedings of a conference on Graph Minors, held at the University of Washington, Seattle, between June 22 and July 5, 1991. The topics of the talks included algorithms on tree-structured graphs, well-quasi-ordering, logic, infinite graphs, disjoint path problems, surface embeddings, knot theory, graph polynomials, matroid theory, and combinatorial optimization. We have tried to organize the volume so that related papers are near one another.

This volume also contains an English translation of a paper by Yves Colin de Verdière, which was first published in a refereed journal in French. It seems worthwhile to make this paper more easily accessible, and we are grateful to the American Mathematical Society and Academic Press for permission to include the translation. All the other papers in this volume have been refereed.

This conference was one in the series of AMS-IMS-SIAM Joint Summer Research Conferences. It was funded by the Office of Naval Research and by the National Science Foundation; we would like to express our thanks to the ONR and NSF for their generous support. Thanks are also due to the other members of the organizing committee: Harvey Friedman and Bruce Reed; Carole Kohanski, who handled the detailed organization of the workshop; and Donna Harmon, Christine Thivierge, and Alison Buckser at the American Mathematical Society, who shepherded us successfully through the publication process.

Neil Robertson
Paul Seymour

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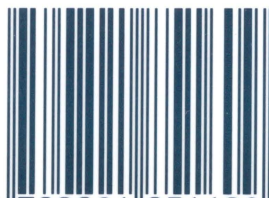
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