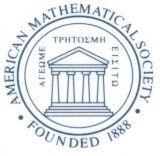
CONTEMPORARY MATHEMATICS

163

Computational Mathematics in China

Zhong-Ci Shi
Chung-Chun Yang
Editors



American Mathematical Society

http://dx.doi.org/10.1090/conm/163

Recent Titles in This Series

163 **Zhong-Ci Shi and Chung-Chun Yang, Editors,** Computational mathematics in China, 1994

162 **Ciro Ciliberto, E. Laura Livorni, and Andrew J. Sommese, Editors,** Classification of algebraic varieties, 1994

161 **Paul A. Schweitzer, S. J., Steven Hurder, Nathan Moreira dos Santos, and José Luis Arraut, Editors,** Differential topology, foliations, and group actions, 1994

160 **Niky Kamran and Peter J. Olver, Editors,** Lie algebras, cohomology, and new applications to quantum mechanics, 1994

159 **William J. Heinzer, Craig L. Huneke, and Judith D. Sally, Editors,** Commutative algebra: Syzygies, multiplicities, and birational algebra, 1994

158 **Eric M. Friedlander and Mark E. Mahowald, Editors,** Topology and representation theory, 1994

157 **Alfio Quarteroni, Jacques Periaux, Yuri A. Kuznetsov, and Olof B. Widlund, Editors,** Domain decomposition methods in science and engineering, 1994

156 **Steven R. Givant,** The structure of relation algebras generated by relativizations, 1994

155 **William B. Jacob, Tsit-Yuen Lam, and Robert O. Robson, Editors,** Recent advances in real algebraic geometry and quadratic forms, 1994

154 **Michael Eastwood, Joseph Wolf, and Roger Zierau, Editors,** The Penrose transform and analytic cohomology in representation theory, 1993

153 **Richard S. Elman, Murray M. Schacher, and V. S. Varadarajan, Editors,** Linear algebraic groups and their representations, 1993

152 **Christopher K. McCord, Editor,** Nielsen theory and dynamical systems, 1993

151 **Matatyahu Rubin,** The reconstruction of trees from their automorphism groups, 1993

150 **Carl-Friedrich Bödigheimer and Richard M. Hain, Editors,** Mapping class groups and moduli spaces of Riemann surfaces, 1993

149 **Harry Cohn, Editor,** Doeblin and modern probability, 1993

148 **Jeffrey Fox and Peter Haskell, Editors,** Index theory and operator algebras, 1993

147 **Neil Robertson and Paul Seymour, Editors,** Graph structure theory, 1993

146 **Martin C. Tangora, Editor,** Algebraic topology, 1993

145 **Jeffrey Adams, Rebecca Herb, Stephen Kudla, Jian-Shu Li, Ron Lipsman, and Jonathan Rosenberg, Editors,** Representation theory of groups and algebras, 1993

144 **Bor-Luh Lin and William B. Johnson, Editors,** Banach spaces, 1993

143 **Marvin Knopp and Mark Sheingorn, Editors,** A tribute to Emil Grosswald: Number theory and related analysis, 1993

142 **Chung-Chun Yang and Sheng Gong, Editors,** Several complex variables in China, 1993

141 **A. Y. Cheer and C. P. van Dam, Editors,** Fluid dynamics in biology, 1993

140 **Eric L. Grinberg, Editor,** Geometric analysis, 1992

139 **Vinay Deodhar, Editor,** Kazhdan-Lusztig theory and related topics, 1992

138 **Donald St. P. Richards, Editor,** Hypergeometric functions on domains of positivity, Jack polynomials, and applications, 1992

137 **Alexander Nagel and Edgar Lee Stout, Editors,** The Madison symposium on complex analysis, 1992

136 **Ron Donagi, Editor,** Curves, Jacobians, and Abelian varieties, 1992

135 **Peter Walters, Editor,** Symbolic dynamics and its applications, 1992

134 **Murray Gerstenhaber and Jim Stasheff, Editors,** Deformation theory and quantum groups with applications to mathematical physics, 1992

133 **Alan Adolphson, Steven Sperber, and Marvin Tretkoff, Editors,** p-Adic methods in number theory and algebraic geometry, 1992

(*Continued in the back of this publication*)

CONTEMPORARY MATHEMATICS

163

Computational Mathematics in China

Zhong-Ci Shi
Chung-Chun Yang
Editors

American Mathematical Society
Providence, Rhode Island

EDITORIAL BOARD

Craig Huneke, managing editor

Clark Robinson — J. T. Stafford

Linda Preiss Rothschild — Peter M. Winkler

1991 *Mathematics Subject Classification.* Primary 62–02; Secondary 01A25.

Papers in this volume are final versions, which will not be published elsewhere.

Library of Congress Cataloging-in-Publication Data

Computational mathematics in China/Zhong-Ci Shi and Chung-Chun Yang, editors.
p. cm. — (Contemporary mathematics; v. 163)
Includes bibliographical references.
ISBN 0-8218-5163-2
1. Numerical calculations. 2. Mathematics—China—Data processing. I. Shih, Chung-tz'u. II. Yang, Chung-Chun, 1942– . III. Series: Contemporary mathematics (American Mathematical Society); v. 163.
QA297.C634 1994 93-45006
519.4—dc20 CIP

Copying and reprinting. Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews, provided the customary acknowledgment of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Manager of Editorial Services, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940-6248. Requests can also be made by e-mail to `reprint-permission@math.ams.org`.

The appearance of the code on the first page of an article in this publication (including abstracts) indicates the copyright owner's consent for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that the fee of $1.00 plus $.25 per page for each copy be paid directly to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, Massachusetts 01923. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotional purposes, for creating new collective works, or for resale.

© Copyright 1994 by the American Mathematical Society. All rights reserved.
The American Mathematical Society retains all rights
except those granted to the United States Government.
Printed in the United States of America.

⊗ The paper used in this book is acid-free and falls within the guidelines established to ensure permanence and durability.

♻ Printed on recycled paper.

This volume was printed from copy prepared by the authors.
This volume was typeset by the authors using AMS-LaTeX,
the American Mathematical Society's TeX macro system.

10 9 8 7 6 5 4 3 2 1 99 98 97 96 95 94

IN MEMORY OF FENG KANG

— The founder and pioneer of China's computational mathematics

Contents

Preface ix

Contributors to This Volume xi

Dynamical Systems and Geometric Construction of Algorithms 1
FENG Kang and WANG Dao-liu

Generalized Stability of Discretization and Its Applications to Numerical Solutions of Nonlinear Differential Equations 33
GUO Ben-yu (KUO Pen-yu)

The Boundary Element Method for Solving Variational Inequalities 55
HAN Hou-de

The Study of Numerical Analysis of Matrix Eigenvalue Problem in China 71
JIANG Er-xiong

Numerical Analysis and Scientific Computation in Hong Kong 77
KWOK Yue-kuen, HUANG Hong-ci and CHAN Raymond H.

Interpolated Finite Elements and Global Error Recovery 93
LIN Qun

Mathematical Theory of Some Non-standard Finite Element Methods 111
SHI Zhong-ci and WANG Ming

Some Results on the Field of Spline Theory and Its Applications 127
SUN Jia-chang

Approximation Theory and Spline Function 139
WANG Ren-hong

A Summary on Continuous Complexity Theory 155
WANG Xing-hua

Viscosity Splitting Schemes 171
YING Long-an

Natural Boundary Element Method and Adaptive Boundary Element Method 185
YU De-hao

Trust Region Algorithms for Nonlinear Programming 205
YUAN Ya-xiang

Preface

Until two to three hundred years ago Chinese mathematics was flourishing, especially in the subject of arithmetic. Examples of its achievements include the calculation of π in the third century and the development of the Chinese Remainder Theorem in the twelfth century. Unfortunately, from this time until the 1950s, Chinese mathematics entered a period of stagnation. A re-awakening came in 1958 when, with the installation of the first Chinese made computer in the Chinese Academy of Sciences, China began to study modern computational mathematics.

Professor Feng Kang played a key role in this development. In the early 1960s his team and he, working in almost complete isolation from the Western World, established the finite element method. In 1965 he wrote a Chinese mathematical paper in which the first rigorous proof of the convergence of the method was given. This was a historically significant contribution to the field of numerical analysis and it was made by a Chinese mathematician. It did not, however, receive timely notice from Western researchers because Professor Feng's paper appeared only in a computational mathematical journal published in China, which ceased publication right after eruption in 1966 of the disastrous 10 year-long Cultural Revolution.

Since 1976, however, the finite element method has regained its momentum in China and impressive progress has been made both in theoretical research of the subject and in its applications. Computational mathematics,too, has rapidly gained a foothold in universities and in government industrial departments all over China. When the great enthusiasm among young students for computational mathematics and the great number of researchers and their abundant results in the subject is compared to the bleak and ignorant situation before 1960, we can proudly say that computational mathematics is one mathematical area that has really taken off in China.

In this collection, twelve key Chinese numerical analysts have presented their important and representative results obtained since the 1960s. The emphasis is on results gained in the last ten years and the topics covered fall into the following fields: finite element method, computational fluid mechanics, numerical

solutions of differential equations, computational methods in dynamical systems, numerical algebra, approximation and optimization. We regret the coverage here may inevitably omit some important works due to the limitations of our energy and time.

We sincerely hope that the publications of this volume will promote further the academic exchange between East and West and allow the Western reader to have a systematic and updated comprehension of the contributions made by Chinese numerical analysts. We believe strongly that through the mutual understanding, common research problems will be identified and further cooperation can be expected. Undoubtedly both East and West will benefit from developments of this kind.

Although the research conducted by Chinese numerical analysts has its own merits, it has also suffered from certain drawbacks. The key problem is that China lags behind in her computing facilities. This has greatly hampered the testing of new algorithms and the implementation of theorems by applying them to practical problems. We are trying to overcome this situation.

When the collection was near to its completion, we were shocked by the news that Professor Feng–the founder and pioneer of China's computational mathematics research passed away suddenly in this August. Before the tragedy, he had been very concerned with the progress of the volume and even contributed an article himself. This is his last article which summarizes his most recent researches and contains many new and stimulating ideas. We regret that he would not witness the publication of this volume.

As the editors of the volume, we want to thank the contributors for their support and cooperation throughout the editing process. We also want to thank American Mathematical Society for its endorsement of the project. Finally, we are grateful to Drs. Wang Dao-Liu and Yu Xi-Jun for their work in preparing the typescript for this volume.

Zhong-Ci Shi 石钟慈
Chung-Chun Yang 杨重骏

Contributors

CHAN, Raymod H. 陈汉夫
Department of Mathematics, Chinese University, Hong Kong

FENG, Kang 冯康
Computing Center, Academia Sinica, Beijing 100080, China

GUO, Ben-yu 郭本瑜
Shanghai University of Science and Technology, Shanghai 201800, China

HAN, Hou-de 韩厚德
Department of Applied Mathematics, Tsinghua University, Beijing 100084, China

HUANG, Hong-ci 黄鸿慈
Department of Mathematics, Hong Kong Baptist College, Hong Kong

JIANG, Er-xiong 蒋尔雄
Department of Mathematics, Fudan University, Shanghai 200433, China

KWOK, Yue-kuen 郭宇权
Department of Mathematics, Hong Kong University of Science and Technology, Hong Kong

LIN, Qun 林群
Institute of System Science, Academia Sinica, Beijing 100080, China

SHI, Zhong-ci 石钟慈
Computing Center, Academia Sinica, Beijing 100080, China

SUN, Jia-chang 孙家昶
Computing Center, Academia Sinica, Beijing 100080, China

WANG, Dao-liu 汪道柳
Computing Center, Academia Sinica, Beijing 100080, China

WANG, Ming 王鸣
Department of Mathematics, Peking University, Beijing 100871, China

WANG, Ren-hong 王仁宏
Institute of Mathematical Sciences, Dalian University of Technology, Dalian 116024, China

WANG, Xing-hua 王兴华
Department of Mathematics, Hangzhou University, Hangzhou 310028, China

YING, Long-an 应隆安
Department of Mathematics, Peking University, Beijing 100871, China

YU, De-hao 余德浩
Computing Center, Academia Sinica, Beijing 100080, China

YUAN, Ya-xiang 袁亚湘
Computing Center, Academia Sinica, Beijing 100080, China

Recent Titles in This Series

(*Continued from the front of this publication*)

132 **Mark Gotay, Jerrold Marsden, and Vincent Moncrief, Editors,** Mathematical aspects of classical field theory, 1992

131 **L. A. Bokut', Yu. L. Ershov, and A. I. Kostrikin, Editors,** Proceedings of the International Conference on Algebra Dedicated to the Memory of A. I. Mal'cev, Parts 1, 2, and 3, 1992

130 **L. Fuchs, K. R. Goodearl, J. T. Stafford, and C. Vinsonhaler, Editors,** Abelian groups and noncommutative rings, 1992

129 **John R. Graef and Jack K. Hale, Editors,** Oscillation and dynamics in delay equations, 1992

128 **Ridgley Lange and Shengwang Wang,** New approaches in spectral decomposition, 1992

127 **Vladimir Oliker and Andrejs Treibergs, Editors,** Geometry and nonlinear partial differential equations, 1992

126 **R. Keith Dennis, Claudio Pedrini, and Michael R. Stein, Editors,** Algebraic K-theory, commutative algebra, and algebraic geometry, 1992

125 **F. Thomas Bruss, Thomas S. Ferguson, and Stephen M. Samuels, Editors,** Strategies for sequential search and selection in real time, 1992

124 **Darrell Haile and James Osterburg, Editors,** Azumaya algebras, actions, and modules, 1992

123 **Steven L. Kleiman and Anders Thorup, Editors,** Enumerative algebraic geometry, 1991

122 **D. H. Sattinger, C. A. Tracy, and S. Venakides, Editors,** Inverse scattering and applications, 1991

121 **Alex J. Feingold, Igor B. Frenkel, and John F. X. Ries,** Spinor construction of vertex operator algebras, triality, and $E_8^{(1)}$, 1991

120 **Robert S. Doran, Editor,** Selfadjoint and nonselfadjoint operator algebras and operator theory, 1991

119 **Robert A. Melter, Azriel Rosenfeld, and Prabir Bhattacharya, Editors,** Vision geometry, 1991

118 **Yan Shi-Jian, Wang Jiagang, and Yang Chung-chun, Editors,** Probability theory and its applications in China, 1991

117 **Morton Brown, Editor,** Continuum theory and dynamical systems, 1991

116 **Brian Harbourne and Robert Speiser, Editors,** Algebraic geometry: Sundance 1988, 1991

115 **Nancy Flournoy and Robert K. Tsutakawa, Editors,** Statistical multiple integration, 1991

114 **Jeffrey C. Lagarias and Michael J. Todd, Editors,** Mathematical developments arising from linear programming, 1990

113 **Eric Grinberg and Eric Todd Quinto, Editors,** Integral geometry and tomography, 1990

112 **Philip J. Brown and Wayne A. Fuller, Editors,** Statistical analysis of measurement error models and applications, 1990

111 **Earl S. Kramer and Spyros S. Magliveras, Editors,** Finite geometries and combinatorial designs, 1990

110 **Georgia Benkart and J. Marshall Osborn, Editors,** Lie algebras and related topics, 1990

109 **Benjamin Fine, Anthony Gaglione, and Francis C. Y. Tang, Editors,** Combinatorial group theory, 1990

108 **Melvyn S. Berger, Editor,** Mathematics of nonlinear science, 1990

107 **Mario Milman and Tomas Schonbek, Editors,** Harmonic analysis and partial differential equations, 1990

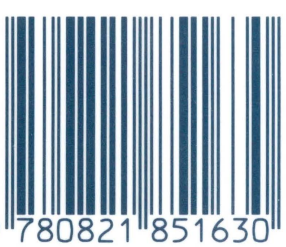
106 **Wilfried Sieg, Editor,** Logic and computation, 1990

105 **Jerome Kaminker, Editor,** Geometric and topological invariants of elliptic operators, 1990

(See the AMS catalog for earlier titles)

Computational Mathematics in China

Zhong-Ci Shi and Chung-Chun Yang, Editors

This volume describes the most significant contributions made by Chinese mathematicians over the past decades in various areas of computational mathematics. Some of the results are quite important and complement Western developments in the field. The contributors to the volume range from noted senior mathematicians to promising young researchers. The topics include finite element methods, computational fluid mechanics, numerical solutions of differential equations, computational methods in dynamical systems, numerical algebra, approximation, and optimization. Containing a number of survey articles, the book provides an excellent way for Western readers to gain an understanding of the status and trends of computational mathematics in China.

ISBN 0-8218-5163-2