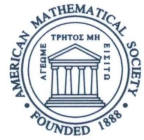
CONTEMPORARY MATHEMATICS

171

Abelian Group Theory and Related Topics

Conference on Abelian Groups
August 1–7, 1993
Oberwolfach, Germany

Rüdiger Göbel
Paul Hill
Wolfgang Liebert
Editors



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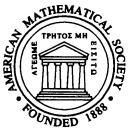

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PREFACE

After an Olympic styled intermission of four years, the Mathematische Forschungsinstitut Oberwolfach reconvened a conference on abelian groups August 1-7, 1993. During a week of beautiful sunshine on the Schwarzwald forest, enlightening talks and bright ideas for new research were emanating from the Institute.

The conference was organized by two local participants, Rüdiger Göbel of Essen and Wolfgang Liebert of München, along with Paul Hill of the United States. It brough together forty-seven participants from all over the world with five continents represented. For the first time in a long series of conferences on abelian groups at Oberwolfach, researchers from Russia and other Eastern bloc countries were in attendance.

The broad geographic scope of the conference yielded world-wide interaction among specialists in abelian groups. Moreover, the conference attracted researchers from other areas of mathematics as well. Indeed, as if reverting to the sixteenth-century concept of the smaller planet being the center of gravity, the conference seemed to have a variety of topics revolving around abelian groups – if only for a week. Experts from model theory, set theory, noncommutative groups, module theory, and from computer science discussed problems in their fields that relate to abelian group theory.

These international conferences on abelian groups have established a tradition of paying tribute to pioneer contributors to the subject such as Prüfer, Ulm, and Baer. This time a more contemporary figure has been selected. We have chosen someone who has been a faithful friend, dedicated advisor, and an inspiring colleague to most who are currently active including the participants of this conference and the contributors to these Proceedings. Certainly, R. S. Pierce was that and more.

Although Pierce had a wide interest in mathematics, he seemed to have a special place in his heart for abelian groups. He made significant contributions to the structure theory of both torsion and torsion-free groups. One of his favorite topics was endomorphisms of abelian groups. After Pierce had become seriously ill, he still made every effort to attend all the major conferences on abelian groups. His interest in what was going on in the subject apparently never waned even over a protracted illness.

The editors invited several people who had a special relationship to R. S. Pierce or his work to contribute papers. We express our appreciation to these and other authors who have made this volume a worthy tribute.

Rüdiger Göbel (Essen)
Paul Hill (Auburn)
Wolfgang Liebert (München)

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Abelian Group Theory and Related Topics
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This volume contains the proceedings of a conference on abelian groups held in August 1993 at Oberwolfach. The conference brought together forty-seven participants from all over the world and from a range of mathematical areas. Experts from model theory, set theory, noncommutative groups, module theory, and computer science discussed problems in their fields that relate to abelian group theory. This book provides a window on the frontier of this active area of research.

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